Applications

Part 1: Examples

Example 1: A ball thrown into the air from a building 100 feet in height travels along a path described by $h(x) = -\frac{x^2}{110} + x + 100$ where h is the ball's height and x is the horizontal distance from the bulding. What is the maximum height the ball will achieve?

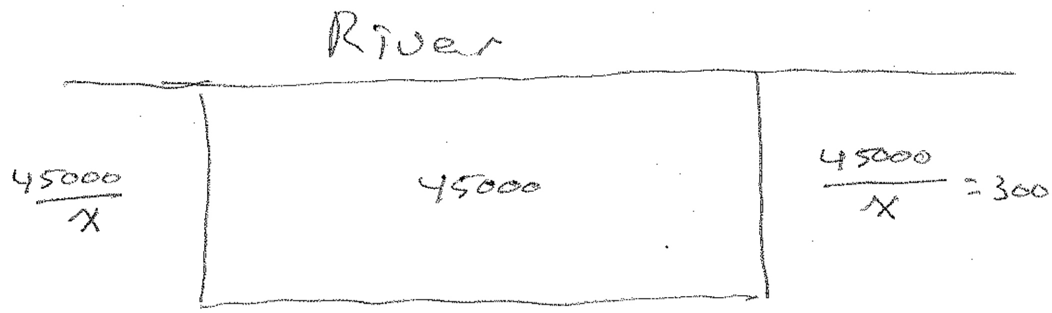
$$h'(x) = -\frac{x}{55} + 1$$

$$\begin{array}{ccc} + & 0 & - \\ \hline & 55 & \end{array}$$

$$h(55) = 127.5$$

The max ht is 127.5 ft.

Example 2: A rectangular field with one side along a river is to be fenced. No fence is needed along the river side. Fence opposite the river costs \$20 per foot while fence on the other sides costs \$5 per foot. Find the minimum cost to fence 45,000 square feet?



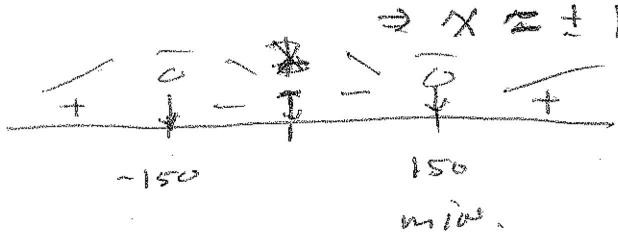
$x = 150$

$$C(x) = 20x + 5 \cdot 2 \cdot \frac{45000}{x}$$

$$C'(x) = 20 - \frac{450000}{x^2}$$

Solve $C' = 0$. $\quad x^2 = \frac{450000}{20}$

$\Rightarrow x = \pm 150$

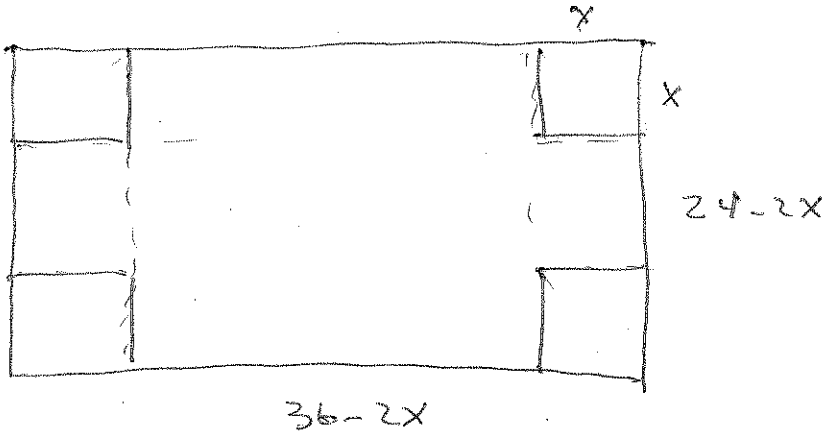


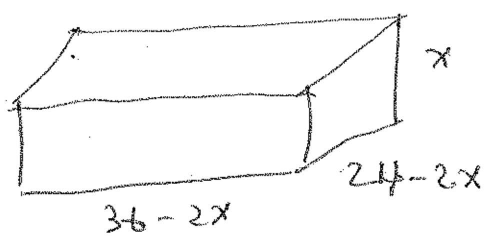
$$C(150) = 20(150) + \frac{450000}{150}$$

$= \$6000$

The min fencing cost is \$6000.

Example 3: A rectangular piece of cardboard is to be used to form a box by cutting squares from the corners. If the original piece is 24 by 36 inches, find the maximum volume that can be enclosed by the box. You may assume the box does not have a top.





$$V(x) = x(24-2x)(36-2x)$$
$$= 864x - 120x^2 + 4x^3$$
$$\Rightarrow V'(x) = 864 - 240x + 12x^2$$
$$= 12(72 - 20x + x^2)$$
$$= 12(12-x)(8-x)$$

V': + 0 − 0 + → x

8 max, 12

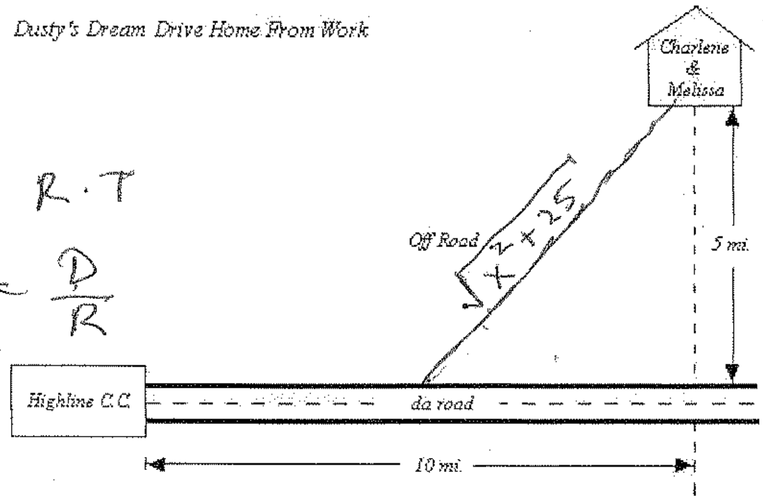
$V(8) = 8(8)(20)$
$= 320$

The max volume is 320 in^3.

Example 4: Dusty wants to get home to his wife Charlene as quickly as possible. His CAT dumptruck can go 30 mph on the road and 20 mph off-road. What is the optimal route for Dusty to take home.

$D = R \cdot T$

$\Rightarrow T = \dfrac{D}{R}$



$|\leftarrow 10 - x \rightarrow|\leftarrow x \rightarrow|$

$$T(x) = (\text{Time on road}) + (\text{Time off road})$$

$$= \frac{10-x}{30} + \frac{\sqrt{x^2+25}}{20}$$

$$\Rightarrow T'(x) = -\frac{1}{30} + \frac{1}{20} \cdot \frac{1}{2}(x^2+25)^{-1/2} \cdot 2x$$

solve $0 = \dfrac{-1}{30} + \dfrac{x}{20\sqrt{x^2+25}}$

$\Rightarrow 20\sqrt{x^2+25} = 30x$

$\Rightarrow x^2 + 25 = \frac{9}{4}x^2$

$\Rightarrow 25 = \frac{5}{4}x^2$

$\Rightarrow 20 = x^2$

$\Rightarrow x = \pm 2\sqrt{5}$

− 0 − 0 +

$\cancel{-2\sqrt{5}}$ extraneous. $2\sqrt{5}$ min.

The best route is on the road for the 1st $(10 - 2\sqrt{5})$ miles & then goes off road.

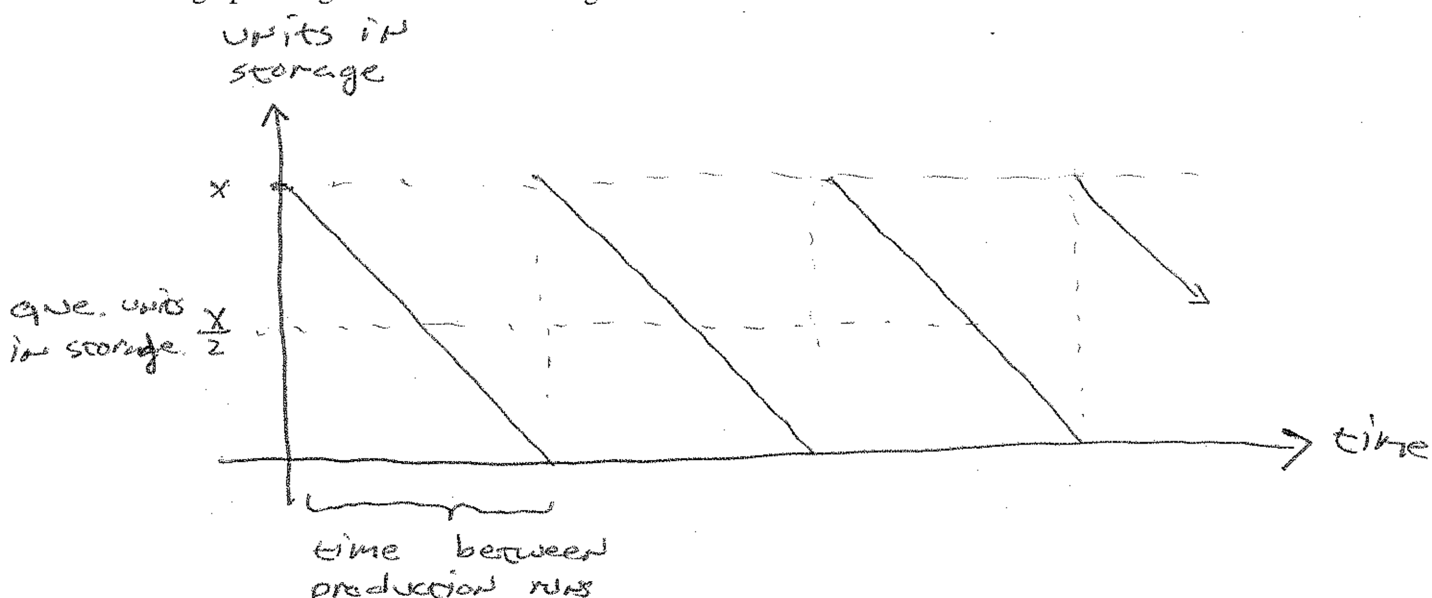
Part 2: Inventory Cost

It costs companies to have inventory on the shelf. (Why is this?) So, many companys produce in what are called production runs. The company goal (and hence our goal) is to minimize the total cost of producing and storing inventory. This requires minimizing a more complicated cost function.

$C =$

$$(\text{number of items}) \cdot (\text{cost}/\text{item}) + (\text{number of runs}) \cdot (\text{cost}/\text{run}) + (\text{average number stored}) \cdot (\text{storage cost}/\text{item})$$

If we assume that x items are produced each run and then they are removed from the shelves at a fixed rate, then a graph that gives the units in storage as a function of time would look like:



In the graph, notice the average units in storage as well as the time between production runs.

Example 5: A company needs 450,000 items per year. Each production run costs \$500 to set up and \$10 for each item produced. It costs \$2 to store an item for up to one year. Find the number of items that should be produced in each run so that the total cost of production and storage is minimized.

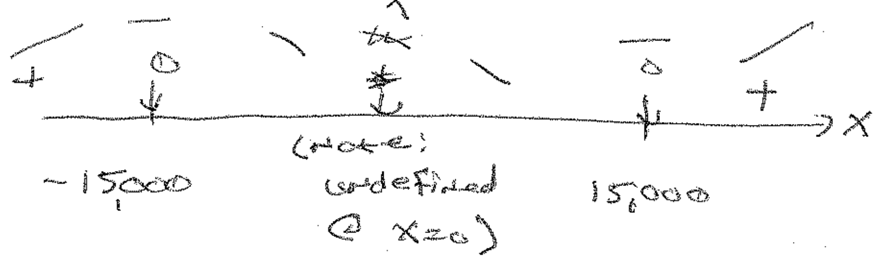
x = # of items produced in each run.

$$C(x) = 450000(10) + \cancel{500(x)}\ 500 \cdot \frac{450000}{x} + \frac{x}{2} \cdot 2$$

$$\cancel{= 4{,}500{,}000 + 501x}$$

$$= 4{,}500{,}000 + \frac{225{,}000{,}000}{x} + x$$

$$\Rightarrow C'(x) = 1 - \frac{225{,}000{,}000}{x^2}$$



Produce 15,000 items in each run.