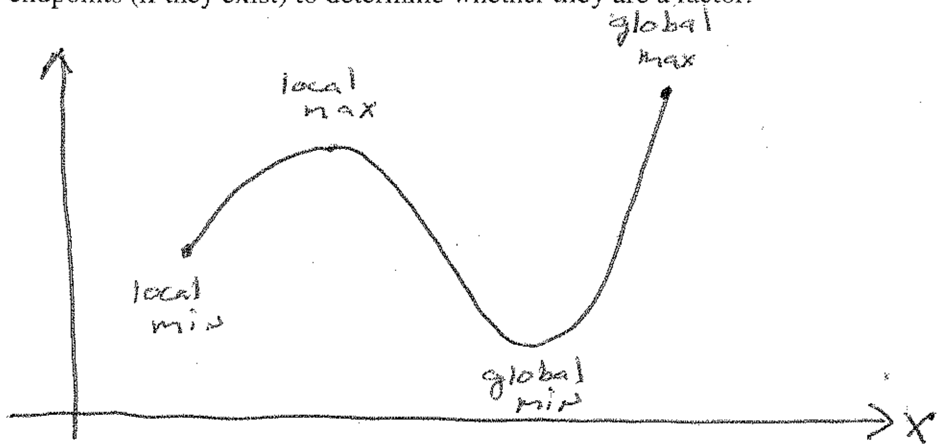
Section 10.03

Relative and Absolute Extremes

Part 1: The picture

To find relative and absolute extremes, you must use the derivative to find "internal" extremes. Then, you must check the endpoints (if they exist) to determine whether they are a factor.



Part 2: Examples

Example 1: Find all extremes of $f(x) = x^3 - x^2 - x$ on $\left[-\frac{1}{2},\ 2\right]$.

$$f'(x) = 3x^2 - 2x - 1$$

$$= (3x+1)(x-1)$$

+	0	−	0	+
	$-\frac{1}{3}$		1	

$y = \frac{1}{8}$	$y = 5/27$	$y = -1$	$y = 2$
min	max	min	max
@	@	@	@
$x = -\frac{1}{2}$	$x = -\frac{1}{3}$	$x = 1$	$x = 2$
local	local	Global	Global

Example 2: A firm has a total revenue given by $R(x) = 2800\,x - 8\,x^2 - x^3$ for a product. Find and interpret the maximum revenue given that at most 25 of the units may be produced.

$$R'(x) = 2800 - 16x - 3x^2$$

$$-100x + 84x$$

$$\cancel{= (28 + x)(84 - 3x)}$$

$$= 100(28 - x) + 3x(28 - x)$$

$$= (100 + 3x)(28 - x)$$

R': $-$ | 0 at $-\frac{100}{3}$ | $+$ | 0 at 28 | $-$ $\to x$

The max rev is $R(25) = \$49375$ when 25 units are sold.

Example 3: If the Math Society charges \$5 admission to a lecture, 100 people will attend. For each \$1 increase in price, 10 fewer people would attend. What price should the Math Society charge to maximize revenue?

\$	#
5	100
6	90
7	80
$5+n$	$100-10n$

$$R(n) = (5+n)(100-10n)$$

$$R'(n) = 1(100-10n) - 10(5+n)$$

$$\cancel{= 100 - 60n - 10n^2}$$

$$\cancel{= -10(-10 + 6n + n^2)}$$

$$= 50 - 20n$$

R': $+$ | 0 at $\frac{5}{2}$ | $-$ $\to n$

Rev. is maxed when \$7.50 is charged/ticket.

Example 4: If the total cost to produce x thousands of units of a product is $C(x) = (x+5)^3$ dollars, find the minimum average cost.

$\cancel{C'(x) = 3(x+5)^2 \cdot 1}$

$\bar{C}'$: $-$ at -5, $-$ at 0, $-$ at $\frac{5}{2}$, $+$ on x

global min.

$$\bar{C}(x) = \frac{(x+5)^3}{x}$$

$$\bar{C}(2.5) = \$168.75$$

$$\bar{C}'(x) = \frac{3(x+5)^2 \cdot x - 1(x+5)^3}{x^2}$$

$$= \frac{(x+5)^2(3x - x + -5)}{x^2}$$

$$= \frac{(x+5)^2(2x-5)}{x^2}$$

Example 4 revisited: If the total cost to produce x thousands of units of a product is $C(x) = (x+5)^3$ dollars, show that the minimum average cost takes place where $\overline{C}(x) = \overline{MC}$.

$$\overline{MC} = 3(x+5)^2$$

$$\bar{C} = \frac{(x+5)^3}{x}$$

solve $3(x+5)^2 = \dfrac{(x+5)^3}{x}$

$\Rightarrow 3x = x+5$

$\Rightarrow 2x = 5$

$\Rightarrow x = \underline{2.5}$ (same as above)

Example 5: A product can be produced for $C(x) = 80000 + 100\,x^2 + x^3$ where x is the number of units produced and sold. The revenue function is $R(x) = \cancel{6000}^{3300}\,x - 50\,x^2$. Find and interpret the maximum profit.

$$P(x) = -50x^2 + \cancel{6000}^{3300}x - x^3 - 100x^2 - 80000$$

$$= -x^3 - 150x^2 + \cancel{6000}^{3300}x - 80000$$

$$P'(x) = -3x^2 - 300x + \cancel{6000}^{3300}$$

$$= -3(x^2 + 100x - \cancel{2000}^{1100})$$

$$= -3(x + 110)(x - 10)$$

$- \quad 0 \quad + \quad 0 \quad -$

$-110 \qquad 10$

profit is maxed when 10 units are sold for a loss of $62,000.

Close shop.

Example 6: A travel agency will plan a tour for groups of 25 or more. It charges $500 per person for a group of 25. However, it charges $10 per person less for each additional member of the group. If it costs the travel agency $125 per member of the group, what size group is the most profitable for the agency?

#	$
25	500
26	490
27	480
⋮	
n	$480 - 10n$

$$R(n) = n(480 - 10n)$$

$$= 480n - 10n^2$$

$$C(n) = 125n$$

$$P(n) = -10n^2 + 355n$$

$$= n(-10n + 355)$$

$$\Rightarrow P'(n) = -20n + 355$$

The most profit is when 43 people go!

$+ \quad 0 \quad -$

$\frac{355}{20}$