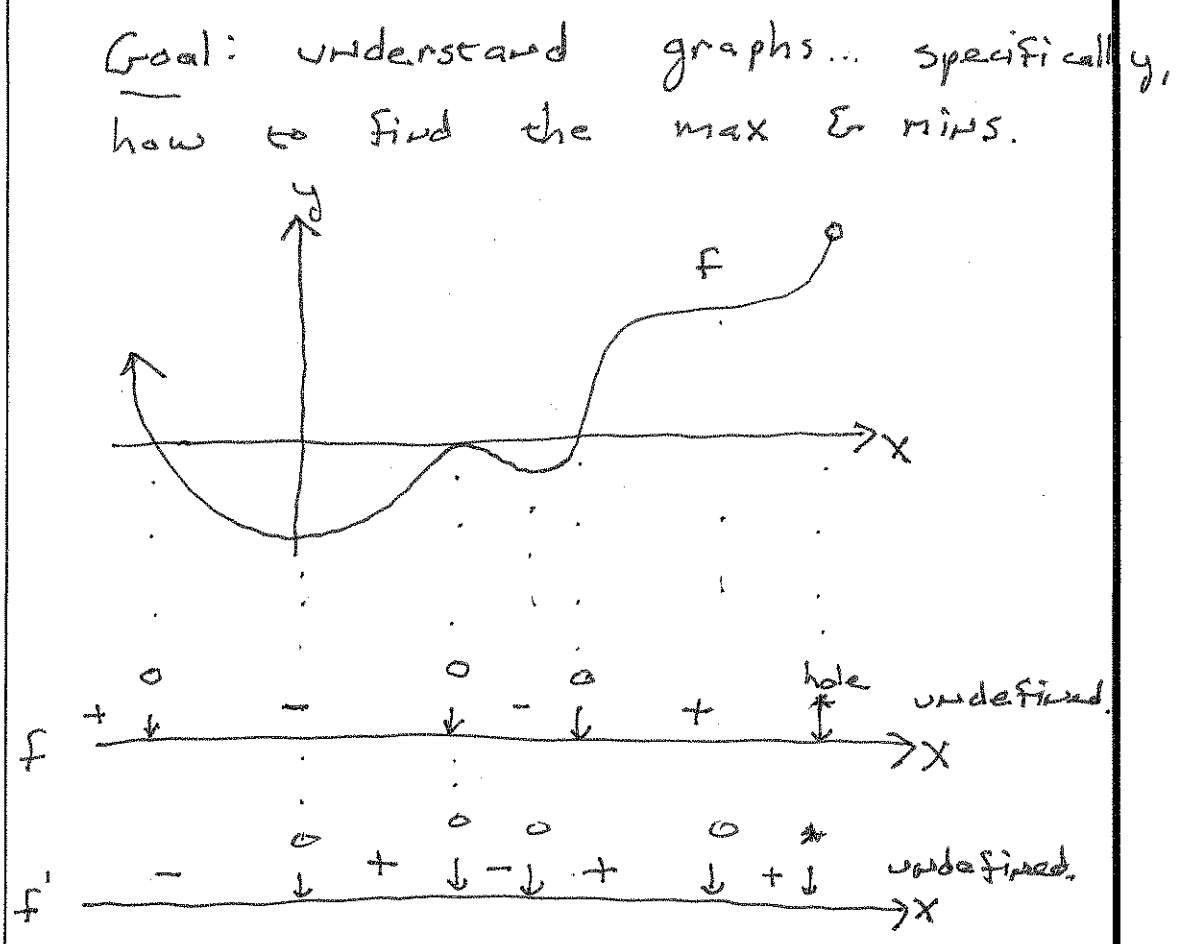
Section 10.01

Curve Sketching

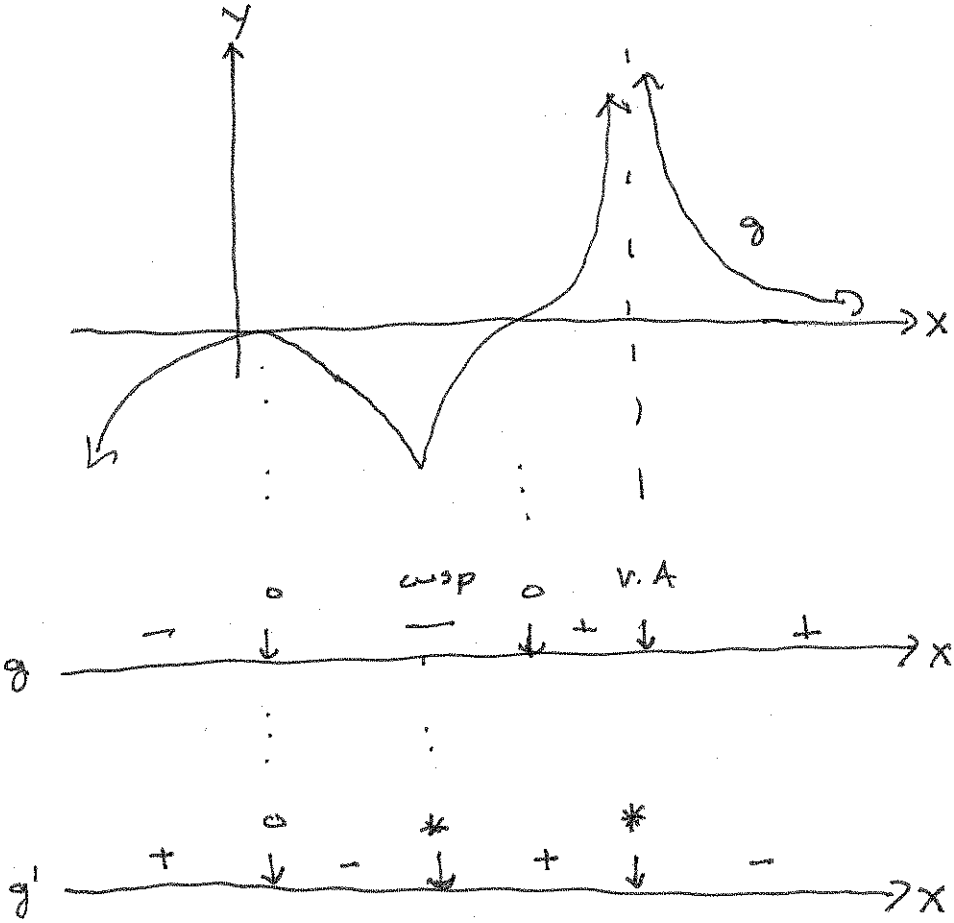
Part 1: Sign Diagrams

Example 1: Construct a sign diagram of a smooth curve

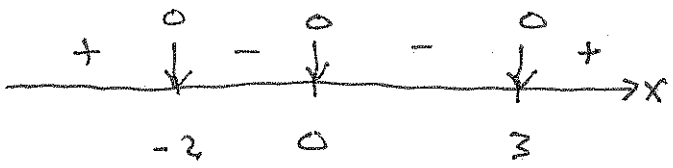


Definition: A critical point or critical value on a graph is a point where the function is discontinuous or has a sharp change in direction. Examples would include (but are not limited to) holes, vertical asymptotes, end points, solitary points, and cusps.

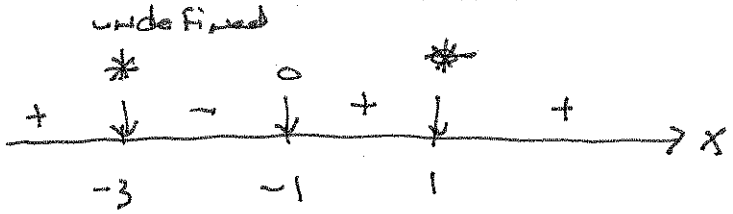
Example 2: Construct a sign diagram of a curve that has a vertical asymptote (V.A.) and cusp (corner).



Example 3: Construct a sign diagram of $y = 3x^2(x+2)(x-3)$



Example 4: Construct a sign diagram of $y = \frac{(x+1)(x-1)}{(x+3)(x-1)}$.



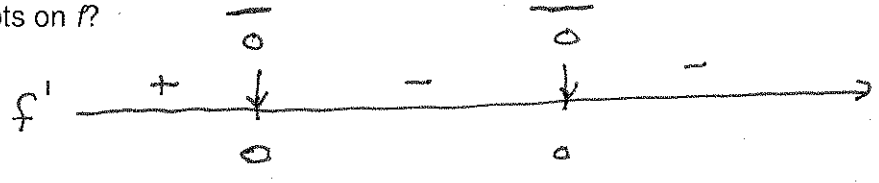
Part 2: Sign Diagrams as a Tool for Curve Sketching

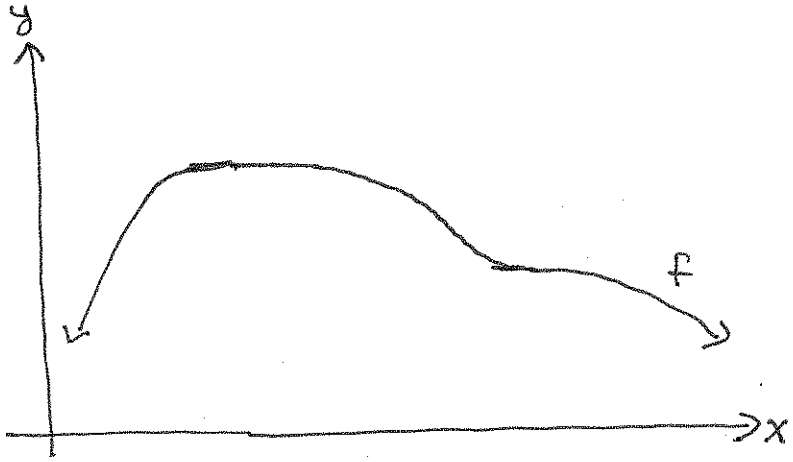
So, where do you see the derivative f' on a graph of f? We see it in the slope of the tangents. That is, when f is increasing, f' will be positive. While, if f is decreasing, f' will be negative. Finally, when f is horizontal (at a max, min, etc.), the derivative will be zero. And, of course there is the possibility that f' will be undefined in some places.

Example 1 revisited: Return to example 1 and sketch a sign diagram of f'.

Example 2 revisited: Return to example 2 and sketch a sign diagram of f'.

Example 5: Given the sign diagram of f', sketch a graph of f. Related question: Does our sign diagram of f' give us any information about the number of x-intercepts on f?

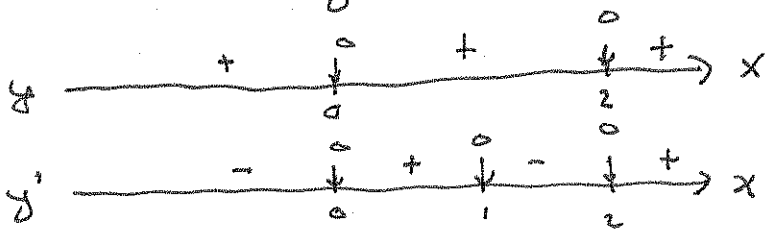


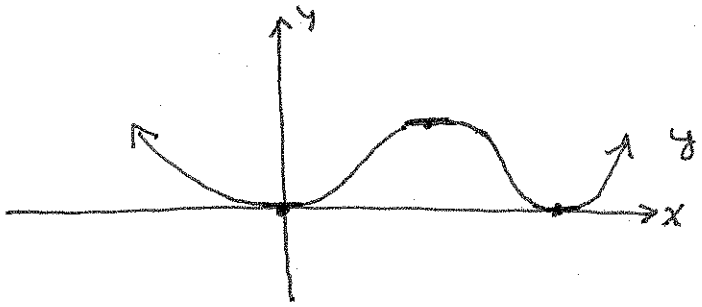


Example 6: Sketch a graph of $y = (x^2 - 2x)^2$ given that $y' = 4x(x-1)(x-2)$

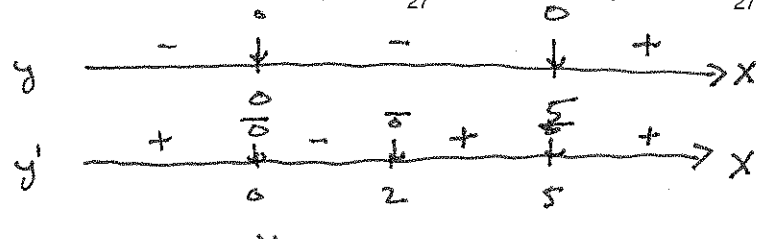
$\Rightarrow y = x^2(x-2)^2$

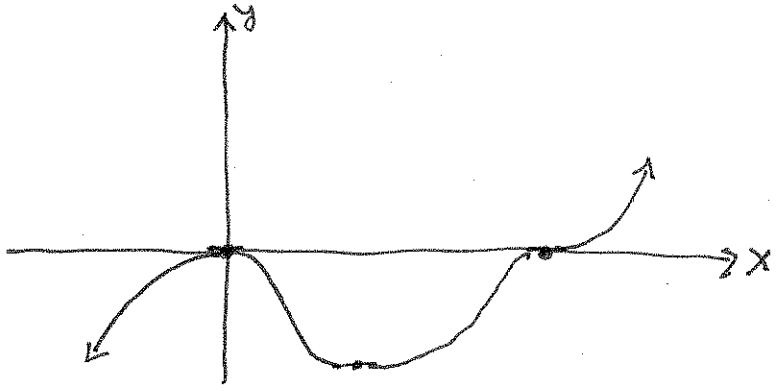
& $y' = 4x(x-1)(x-2)$





Example 7: Sketch a graph of $y = \frac{x^2(x-5)^3}{27}$ given that $y' = \frac{5x(x-2)(x-5)^2}{27}$



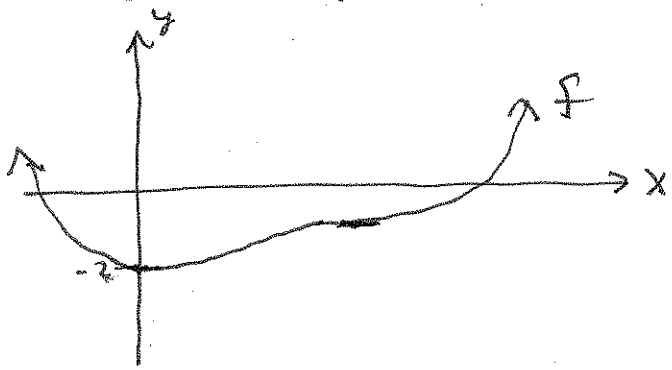


Example 8: Sketch a graph of $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - 2$

$f'(x) = x^3 - 2x^2 + x$

$= x(x^2 - 2x + 1)$

$= x(x-1)^2$



Example 9: Sketch a graph of $g(x) = 3x^5 - 5x^3 + 1$

$\Rightarrow g'(x) = 15x^4 - 15x^2$

$= 15x^2(x^2 - 1)$

$= 15x^2(x+1)(x-1)$