

## The Product and Quotient Rules

## Part 1: The Product Rule

**Example 1:** How would we approach  $\frac{d}{dx}(3x^4 - 2x^3 + 7)(x^3 + 2x^2 + 4x - 9)$  using the techniques of the previous section?

We would expand (a pain) and then use the power rule.



**Derivative Rule: The Product Rule**

If  $f(x) = u(x) \cdot v(x)$ , where  $u$  and  $v$  are differentiable functions of  $x$ , then  
 $f'(x) = u'(x) \cdot v(x) + v'(x) \cdot u(x)$ .

This can be memorized as,  $(u \cdot v)' = u'v + v'u$ .

**Example 1 revisited:** Find  $\frac{d}{dx}(3x^4 - 2x^3 + 7)(x^3 + 2x^2 + 4x - 9)$  using the product rule (Hint: What are  $u$  and  $v$ ?)

$$u = 3x^4 - 2x^3 + 7 \qquad u' = 12x^3 - 6x^2$$

$$v = x^3 + 2x^2 + 4x - 9 \qquad v' = 3x^2 + 4x + 4$$

$$(uv)' = (12x^3 - 6x^2)(x^3 + 2x^2 + 4x - 9) + (3x^2 + 4x + 4)(3x^4 - 2x^3 + 7)$$

**Example 2:** Find  $y'$  if  $y = (x+3)(x^2 - 2x)$ .

$$y' = 1(x^2 - 2x) + (2x - 2)(x + 3)$$

**Example 3 (for you):** Find  $\frac{ds}{dt}$  if  $s = (t^2 + 1)(t^3 - 1)$

$$\frac{ds}{dt} = (2t)(t^3 - 1) + (3t^2)(t^2 + 1)$$

**Example 4:** Differentiate  $f(x) = (7x^6 - 5x^4 + 2x^2 - 1)(4x^9 + 3x^7 - 5x^2 + 3x)$ , but do not simplify your result.

$$f'(x) = (42x^5 - 20x^3 + 4x)(4x^9 + 3x^7 - 5x^2 + 3x) + (36x^5 + 21x^6 - 10x + 3)(7x^6 - 5x^4 + 2x^2 - 1)$$

**Example 5:** Differentiate  $y = (\sqrt[5]{x} - 2\sqrt[4]{x} + 1)(x^3 - \frac{5}{x} + 3x^{-8})$ , but do not simplify your result.

$$y' = \left( \frac{1}{5} x^{-4/5} - \frac{2}{4} x^{-3/4} \right) \left( x^3 - \frac{5}{x} + 3x^{-8} \right) + \left( 3x^2 + \frac{5}{x^2} - 24x^{-9} \right) \left( x^{1/5} - 2x^{1/4} + 1 \right)$$

**Example 6:** An agency will give tours for groups of at least 25. The cost is \$300/person for groups of 25. The cost is reduced by \$10 (per person) for each person over 25.

$N = \#$  of people...

a.) Find  $R(n)$  (the revenue function)

$$R(N) = N \left( \underbrace{300 - 10(N - 25)}_{300 - 10N + 250} \right), N = 25, 26, \dots$$

$$= 550N - 10N^2$$

b.) Find  $D_R$  (the domain)

$$D_R = \{N \mid N = 25, 26, \dots\}$$

alt. sol.  
if  $N = \#$  above 25  
 $R(N) = (25 + N)(300 - 10N)$ .

c.) Find  $\overline{MR}$

$$\overline{MR} = 1(550 - 10N) + N(-10)$$

As the group size increases from 30 to 31, the total revenue <sup>above.</sup> decreases by \$50.

d.) Find and interpret  $\overline{MR}(30)$

$$\overline{MR}(30) = 550 - 10(30) - 10(30) = -50$$

## Part 2: The Quotient Rule



**Derivative Rule:** The quotient rule

If  $f(x) = \frac{u(x)}{v(x)}$ , where  $u$  and  $v$  are differentiable functions of  $x$  with  $v(x) \neq 0$ , then

$$f'(x) = \frac{u'(x)v(x) - v'(x)u(x)}{[v(x)]^2}.$$

This can be memorized as,  $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$ .

**Example 7:** If  $f(x) = \frac{x^3+2x}{x^2-7}$ , find  $f'$

$$u = x^3 + 2x \quad u' = 3x^2 + 2$$

$$v = x^2 - 7 \quad v' = 2x$$

$$f'(x) = \frac{(3x^2+2)(x^2-7) - 2x(x^3+2x)}{(x^2-7)^2}$$

**Example 8:** Find  $\frac{d}{dx} \frac{1-2x^2}{x^4-2x^2+5}$

$$= \frac{-4x(x^4-2x^2+5) - (4x^3-4x+5)(1-2x^2)}{(x^4-2x^2+5)^2}$$

**Quiz - Just for you**

a.) Write down the product rule

$$(u \cdot v)' = u'v + v'u$$

b.) Write down the quotient rule

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

c.) What two pieces of information do you need to find the equation of a tangent line? And, what formula is most helpful for writing the equation of the line?

pt & slope are the info needed.  
then use pt-slope form!  $y - y_1 = m(x - x_1)$

**Example 9:** Find the equation of the tangent line to  $y = (4x^2 + 4x + 1)(7 - 2x)$  when  $x = 1$ .

$$y' = (8x + 4)(7 - 2x) + (-2)(4x^2 + 4x + 1)$$

$$@ x = 1: y' = 12(5) - 2(9) = 42$$

$$y @ x = 1: y = (9)(5) = 45$$

$$y - 45 = 42(x - 1).$$

Example 10: Experimental evidence has shown that the concentration of injected adrenaline  $x$  is related to the response  $y$  of a muscle according to the equation  $y = \frac{x}{a+bx}$  where  $a$  and  $b$  are constants. Find the ROC of response with respect to (WRT) concentration.

$$y' = \frac{1(a+bx) - b(x)}{(a+bx)^2}$$