

Review Exercises

CH 12 Evaluate the integrals in Problems 1–26.

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| 1. $\int x^6 dx$ | 2. $\int x^{1/2} dx$ |
| 3. $\int (12x^3 - 3x^2 + 4x + 5) dx$ | 4. $\int 7(x^2 - 1)^2 dx$ |
| 5. $\int 7x(x^2 - 1)^2 dx$ | 6. $\int (x^3 - 3x^2)^5(x^2 - 2x) dx$ |
| 7. $\int (x^3 + 4)^2 3x dx$ | 8. $\int 5x^2(3x^3 + 7)^6 dx$ |
| 9. $\int \frac{x^2}{x^3 + 1} dx$ | 10. $\int \frac{x^2}{(x^3 + 1)^2} dx$ |
| 11. $\int \frac{x^3 dx}{\sqrt[3]{x^3 - 4}}$ | 12. $\int \frac{x^2 dx}{x^3 - 4}$ |
| 13. $\int \frac{x^3 + 1}{x^2} dx$ | 14. $\int \frac{x^3 + 3x + 1}{x^2} dx$ |
| 15. $\int y^2 e^{y^3} dy$ | 16. $\int (3x - 1)^{12} dx$ |
| 17. $\int \frac{3x^2}{2x^3 - 7} dx$ | 18. $\int \frac{5 dx}{e^{4x}}$ |
| 19. $\int (x^3 - e^{3x}) dx$ | 20. $\int xe^{1+x^2} dx$ |
| 21. $\int \frac{6x^7}{(5x^8 + 7)^3} dx$ | 22. $\int \frac{7x^3}{\sqrt{1-x^4}} dx$ |
| 23. $\int \left(\frac{e^{2x}}{2} + \frac{2}{e^{2x}} \right) dx$ | 24. $\int \left[x - \frac{1}{(x+1)^2} \right] dx$ |

35. **Revenue** If the marginal revenue for a month for a product is $\overline{MR} = 0.06x + 12$ dollars per unit, find the total revenue from the sale of $x = 800$ units of the product.

40. **Revenue** If the marginal revenue for a product is $\overline{MR} = \frac{800}{x+2}$, find the total revenue function.

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| 25. (a) $\int (x^2 - 1)^4 x dx$ | (b) $\int (x^2 - 1)^{10} x dx$ |
| (c) $\int (x^2 - 1)^7 3x dx$ | (d) $\int (x^2 - 1)^{-2/3} x dx$ |
| 26. (a) $\int \frac{2x dx}{x^2 - 1}$ | (b) $\int \frac{2x dx}{(x^2 - 1)^2}$ |
| (c) $\int \frac{3x dx}{\sqrt{x^2 - 1}}$ | (d) $\int \frac{3x dx}{x^2 - 1}$ |

41. **Cost** The marginal cost for a product is $\overline{MC} = 6x + 4$ dollars per unit, and the cost of producing 100 items is \$31,400.

- (a) Find the fixed costs.
 (b) Find the total cost function.

42. **Profit** Suppose a product has a daily marginal revenue $\overline{MR} = 46$ and a daily marginal cost $\overline{MC} = 30 + \frac{1}{3}x$, both in dollars per unit. If the daily fixed cost is \$200, how many units will give maximum profit and what is the maximum profit?

43. **National consumption** If consumption is \$8.5 billion when disposable income is 0, and if the marginal propensity to consume is

$$\frac{dC}{dy} = \frac{1}{\sqrt{2y + 16}} + 0.6 \quad (\text{in billions of dollars})$$

find the national consumption function.

44. **National consumption** Suppose that the marginal propensity to save is

$$\frac{dS}{dy} = 0.2 - 0.1e^{-2y} \quad (\text{in billions of dollars})$$

and consumption is \$7.8 billion when disposable income is 0. Find the national consumption function.

CH 13 Evaluate the integrals in Problems 7–18.

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| 7. $\int_1^4 4\sqrt{x^3} dx$ | 8. $\int_{-3}^2 (x^3 - 3x^2 + 4x + 2) dx$ |
| 9. $\int_0^5 (x^3 + 4x) dx$ | 10. $\int_{-1}^3 (3x + 4)^{-2} dx$ |
| 11. $\int_{-3}^{-1} (x + 1) dx$ | 12. $\int_2^3 \frac{x^2}{2x^3 - 7} dx$ |
| 13. $\int_{-1}^2 (x^2 + x) dx$ | 14. $\int_1^4 \left(\frac{1}{x} + \sqrt{x} \right) dx$ |
| 15. $\int_0^2 5x^2(6x^3 + 1)^{1/2} dx$ | 16. $\int_0^1 \frac{x}{x^2 + 1} dx$ |
| 17. $\int_0^1 e^{-2x} dx$ | 18. $\int_0^1 xe^{x^2} dx$ |

Find the area between the curves in Problems 19–22.

19. $y = x^2 - 3x + 2$ and $y = x^2 + 4$ from $x = 0$ to $x = 5$
 20. $y = x^2$ and $y = 4x + 5$
 21. $y = x^3$ and $y = x$ from $x = -1$ to $x = 0$
 22. $y = x^3 - 1$ and $y = x - 1$

39. **Maintenance** Maintenance costs for buildings increase as the buildings age. If the rate of increase in maintenance costs for a building is

$$M'(t) = \frac{14,000}{\sqrt{t + 16}}$$

where M is in dollars and t is time in years, $0 \leq t \leq 15$, find the total maintenance cost for the first 9 years ($t = 0$ to $t = 9$).

46. *Income streams* Find the total income over the next 10 years from a continuous income stream that has an annual flow rate at time t given by $f(t) = 125e^{0.05t}$ in thousands of dollars per year.

47. *Income streams* Suppose that a machine's production is considered a continuous income stream with an annual rate of flow at time t given by $f(t) = 150e^{-0.2t}$ in thousands of dollars per year. Money is worth 8%, compounded continuously.

(a) Find the present value of the machine's production over the next 5 years.

(b) Find the future value of the production 5 years from now.

48. *Average cost* Suppose the cost function for x units of a product is given by $C(x) = \sqrt{40,000 + x^2}$ dollars. Find the average cost over the first 150 units.

49. *Producer's surplus* Suppose the supply function for x units of a certain lamp is given by

$$p = 0.02x + 50.01 - \frac{10}{\sqrt{x^2 + 1}}$$

where p is in dollars. Find the producer's surplus if the equilibrium price is \$70 and the equilibrium quantity is 1000.

50. *Income streams* Suppose the present value of a continuous income stream over the next 5 years is given by

$$P = 9000 \int_0^5 te^{-0.08t} dt, \quad P \text{ in dollars, } t \text{ in years}$$

Find the present value.

51. *Cost* If the marginal cost for x units of a product is $\overline{MC} = 3 + 60(x + 1) \ln(x + 1)$ dollars per unit and if the fixed cost is \$2000, find the total cost function.

53. *Capital value* Find the capital value of a business if its income is considered a continuous income stream with annual rate of flow given by

$$f(t) = 120e^{0.03t}$$

in thousands of dollars per year, and the current interest rate is 6% compounded continuously.

54. *Total income* Suppose that a continuous income stream has an annual rate of flow $f(t) = 100e^{-0.01t^2}$ (in thousands of dollars per year). Use Simpson's Rule with $n = 4$ to approximate the total income from this stream over the next 2 years.

41. *Savings* The future value of \$1000 invested in a savings account at 10%, compounded continuously, is $S = 1000e^{0.1t}$, where t is in years. Find the average amount in the savings account during the first 5 years.

42. *Income streams* Suppose the total income in dollars from a video machine is given by

$$I = 50e^{0.2t}, \quad 0 \leq t \leq 4, t \text{ in hours}$$

Find the average income over this 4-hour period.

43. *Income distribution* In 1969, after the "Great Society" initiatives of the Johnson administration, the Lorenz curve for the U.S. income distribution was $L(x) = x^{2.1936}$. In 2000, after the stock market's historic 10-year growth, the Lorenz curve for the U.S. income distribution was $L(x) = x^{2.4870}$. Find the Gini coefficient of income for both years, and determine in which year income was more equally distributed.

44. *Consumer's surplus* The demand function for a product under pure competition is $p = \sqrt{64 - 4x}$, and the supply function is $p = x - 1$, where x is the number of units and p is in dollars.

(a) Find the market equilibrium.

(b) Find the consumer's surplus at market equilibrium.

45. *Producer's surplus* Find the producer's surplus at market equilibrium for Problem 44.