Test 1
Dusty Wilson
Math 148

Name: _____**KEY**_____

*It is rare to find learned men who are clean, do
not stink and have a sense of humour.*

No work = no credit

Duchess of Orléans

Warm-ups (1 pt each): $(-3)^2 = \underline{9}$ $-3^2 = \underline{-9}$ $\frac{2}{0} = \underline{und.}$

1.) (1 pt) The quote above is attributed to the Duchess of Orléans regarding Gottfried Leibniz (the co-discover/inventor of calculus). What special qualities did Leibniz have? Use complete English sentences.

Leibniz took baths.

2.) (16 pts) Find the derivatives of the following: (Simplification is optional).

a.) (4 pts) $y = 3 - 5x^8 - \frac{1}{x^2} + \sqrt[4]{x} + \frac{1}{x}$

$-x^{-2} + x^{1/4} - x^{-1}$

$y' = -40x^7 + x^{-3} + \frac{1}{4}x^{-3/4} + x^{-2}$

b.) (4 pts) $y(x) = (3x^5 + x^7 - 2x) \cdot (5 - 4x^2)$

$y'(x) = (15x^4 + 7x^6 - 2)(5 - 4x^2) + (-8x)(3x^5 + x^7 - 2x)$

c.) (4 pts) $z = \frac{3x^2 + x}{2x^4 + x^3}$

$\frac{dz}{dx} = \frac{(6x+1)(2x^4 + x^3) - (8x^3 + 3x^2)(3x^2 + x)}{(2x^4 + x^3)^2}$

d.) (4 pts) $f(x) = 8\sqrt{4x^5 - 3x}$ $= 8(4x^5 - 3x)^{1/2}$

$f'(x) = 8 \cdot \frac{1}{2}(4x^5 - 3x)^{-1/2} \cdot (20x^4 - 3)$

3.) (4 pts) Find $f''(x)$ if $f(x) = \sqrt[3]{x} - 7x^4$.

$$= x^{1/3} - 7x^4$$

$$f'(x) = \frac{1}{3}x^{-2/3} - 28x^3$$

$$f''(x) = -\frac{2}{9}x^{-5/3} - 74x^2$$

4.) (4 pts) If $f(n) = \underbrace{(n - n^3)}_{u} \cdot \underbrace{(4 + 5n^2)^7}_{v}$, find $\dfrac{df}{dn}$

$$f'(n) = (1 - 3n^2)(4 + 5n^2)^7 + 7(4 + 5n^2)^6 \cdot (10n)(n - n^3)$$

5.) (5 pts) Given the function $f(x)$ shown in the graph to the right, evaluate the following:

a.) $f(-1) = \neq 2$

b.) $\lim\limits_{x \to -1^+} f(x) = 2$

c.) $\lim\limits_{x \to -1^-} f(x) = 1$

d.) $\lim\limits_{x \to -1} f(x) = DNE$

e.) Is $f(x)$ continuous at $x = 1$?
Explain why or why not using the <u>definition of continuity</u>?
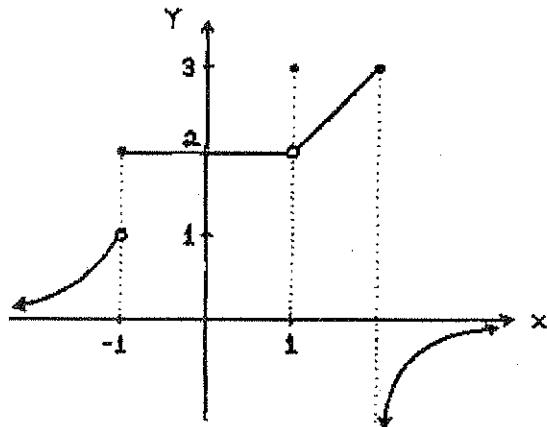
No. $f(1) = 3 \neq 2 = \lim\limits_{x \to 1} f(x)$.

6. (3 pt) Consider the table of values for $y = f(x)$. Suppose that $f(2)$ is undefined.

a.) Estimate $\lim\limits_{x \to 2} f(x)$

8

b.) Based upon the given information, is $f(x)$ continuous at $x = 2$?
Explain your answer.

No, $f(2)$ is undefined.

x	$f(x)$
1.9	6.849
1.99	7.8805
1.999	7.988
$\vdots$	$\vdots$
2	undefined
$\vdots$	$\vdots$
2.001	8.012
2.01	8.1205
2.1	9.251

c.) Use the table to calculate the average rate of change of $f(x)$ on the interval $[1.99, 2.10]$

$$\text{Ave Roc} = \frac{f(2.10) - f(1.99)}{2.10 - 1.99} = \frac{8.1205 - 7.8805}{.11} = 2.18$$

7.) (4 pts) Use the <u>definition of the derivative</u> to find $f'(x)$ if $f(x) = 3x - 5x^2$

$$f'(x) = \lim_{h \to 0} \frac{\left[3(x+h) - 5(x+h)^2\right] - \left[3x - 5x^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h - 5(x^2 + 2xh + h^2) - 3x + 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{3h - 5x^2 - 10xh - 5h^2 + 5x^2}{h}$$

$$= \lim_{h \to 0} (3 - 10x - 5h)$$

$$= 3 - 10x$$

$$\underline{f'(x) = 3 - 10x}$$

8.) (4 pts) Consider the function: $g(x) = \dfrac{3x - 6}{2x - 4} = \dfrac{3(x-2)}{2(x-2)}$

a.) $g(2) = $ _____ und. _____

b.) $\lim\limits_{x \to 2} g(x) = $ _____ $3/2$ _____

c.) $\lim\limits_{x \to -\infty} g(x) = $ _____ $3/2$ _____

d.) $\lim\limits_{x \to +\infty} g(x) = $ _____ $3/2$ _____

9.) (7 pts) The cost C in dollar from the production of x of the all new iPad's is:

$$C(x) = 400x - 0.01x^2 + 19,000,000$$
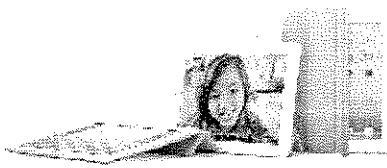
a.) (1 pt) Find and interpret $C(0)$.

The fixed costs are $19,000,000

b.) (2 pts) Calculate $\overline{MC}(x)$. $= 400 - 0.02x$

$$\overline{MC}(x) = 400 - 0.02x$$

c.) (2 pt) Interpret $\overline{MC}(10,000)$ using complete English sentences.

The cost of producing the 10,000th item is about $200.

d.) (2 pts) Find and interpret $C(10,000) - C(9,999)$ using complete English sentences.

The exact cost of producing the 10,000th item is $200.01

10.) (4 pts) Consider the function $g(x) = x^2$.

a.) Find $g(3.1)$

$$g(3.1) = 9.61$$

b.) Find the equation of the tangent line when $x = 3$

$g'(x) = 2x \big|_{x=3}$ 6 (slope)

$g(3) = 9$ (pt)

$$y - 9 = 6(x - 3)$$

c.) Find the y value on the tangent line from (b.) corresponding to $x = 3.1$.

$$y - 9 = (6)[(3.1) - 3)] \Rightarrow y = 9.6$$

d.) Compare the results of (a.) and (c.). and explain the similarity of the results.

The results are similar because the tangent line approxs $g(x)$ near $x=3$.