**Test 1**
Dusty Wilson
Math 148

**No work = no credit**

**Name**: KEY

*Young men should prove theorems, old men should write books.*

Godfrey Harold "GH" Hardy (1877 - 1947)
English mathematician

Warm-ups (1 pt each): $(-2)^2 =$ 4 $\quad -2^2 =$ -4 $\quad \frac{2}{0} =$ UNDEFINED

1.) (1 pt) Paraphrase the quote by GH Hardy given above. Use complete English sentences.

Be creative while you are young

2.) (16 pts) Find the derivatives of the following: (Simplification is optional).

a.) (4 pts) $y = 4x^6 - \frac{1}{x} + \sqrt{x} + 4$ $\quad = 4x^6 - x^{-1} + x^{1/2} + 4$

$y' = 24x^5 + x^{-2} + \frac{1}{2}x^{-1/2}$

b.) (4 pts) $f(x) = \frac{1}{7}(2x^3 - x)^7$

$f'(x) = (2x^3 - x)^6 \cdot (6x^2 - 1)$

c.) (4 pts) $y(x) = (2x^4 + x^2 - 7)\cdot(5x - x^3)$

$y'(x) = (8x^3 + 2x)(5x - x^3) + (5 - 3x^2)(2x^4 + x^2 - 7)$

d.) (4 pts) $z = \frac{4x^2}{3x^5 + 8}$

$$z' = \frac{8x(3x^5 + 8) - 15x^4 \cdot 4x^2}{(3x^5 + 8)^2}$$



$\overline{20}$ $\qquad \overline{53}$

3.) (4 pts) Find $f''(x)$ if $f(x)=2x^5-\sqrt{x}$.

$$=2x^5 - x^{1/2}$$

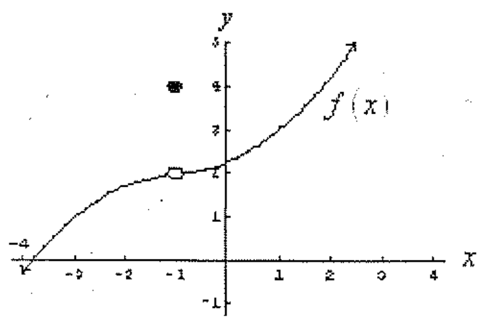
$$\Rightarrow f'(x) = 10x^4 - \frac{1}{2}x^{-1/2}$$

$$f''(x) = 40x^3 + \frac{1}{4}x^{-3/2}$$

4.) (4 pts) If $f(q)=q^4\cdot(q^2+5)^7$, find $\frac{df}{dq}$

$$\frac{df}{dq} = 4q^3\cdot(q^2+5)^7 + 7(q^2+5)^6\cdot 2q\cdot q^4$$

5.) (5 pts) Given the function $f(x)$ shown in the graph to the right, evaluate the following:




a.) $f(-1)=$ 4

b.) $\lim_{x\to -1^+} f(x) =$ 2

c.) $\lim_{x\to -1^-} f(x)=$ 2

d.) $\lim_{x\to -1} f(x)=$ 2

e.) Is $f(x)$ continuous? Explain why or why not using the <u>definition of continuity</u>?

No $\rightarrow$ $f(-1) \neq \lim_{x\to -1} f(x)$



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6.) (1 pt) Find the derivative of $f(x)=1+\frac{x}{1}+\frac{x^2}{1\cdot 2}+\frac{x^3}{1\cdot 2\cdot 3}+\frac{x^4}{1\cdot 2\cdot 3\cdot 4}+\cdots$

Express your answer in terms of $f(x)$.

$$f'(x) = 1 + \frac{x}{1} + \frac{x^2}{1\cdot 2} + \frac{x^3}{1\cdot 2\cdot 3} + \ldots$$

$$\Rightarrow \quad f(x) = f'(x)$$

7.) (4 pts) Use the definition of the derivative to find $g'(x)$ if $g(x)=3x^2-5$

$$g'(x) = \lim_{h\to 0} \frac{[3(x+h)^2 - 5] - (3x^2 - 5)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - 5 - \cancel{3x^2} + 5}{h}$$

$$= \lim_{h\to 0} (6x + 3h)$$

$$= 6x$$

$g'(x) = 6x$

8.) (4 pts) Consider the function: $g(x)=\frac{x^2+2x+1}{x^2+5x+4}$ $= \frac{(x+1)^2}{(x+4)(x+1)}$

a.) $g(-1)=$ undefined

b.) $\lim_{x\to -1} g(x)=$ 0

c.) $\lim_{x\to -\infty} g(x)=$ 1

d.) $\lim_{x\to +\infty} g(x)=$ 1



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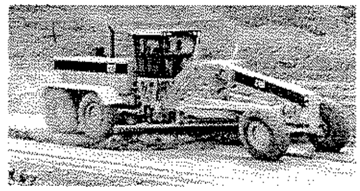
9.) (7 pts) The revenue $R$ ~~in thousands~~ of dollar from the sale of $x$ Caterpillar Motor Graders is given by

$$R(x) = 15x^4 + 450x^3$$



Caterpillar
Motor Grader 24H

a.) (1 pt) Find and interpret $R(0)$. $= 0$

No revenue is generated when No Motor Grader 24 H's are sold.

b.) (2 pts) Calculate $\overline{MR}(x)$.

$\overline{MR}(x) = 60x^3 + 1350x^2$

c.) (2 pt) Interpret $\overline{MR}(20)$ using complete English sentences.

The revenue from the sale of the 20th motor grader is about \$1,020,000

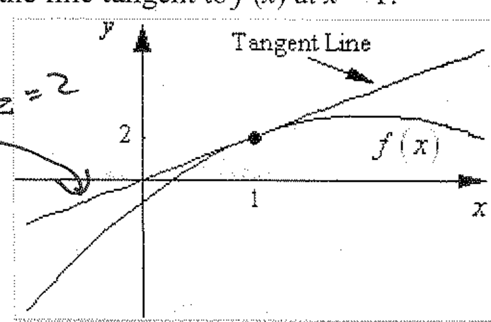
d.) (2 pts) Find and interpret $\underbrace{R(21) - R(20)}_{1,084,665}$ using complete English sentences.

The exact revenue from the 21st item is about \$1,084,665 (or \$958,635 for the 20th)

10.) (4 pts) The graph shown to the right portrays $f(x)$ and the line tangent to $f(x)$ at $x = 1$.

Use this information to find $f'(1)$

slope = 2




$f'(1) =$ 2



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