

Test 1
Dusty Wilson
Math 148

Name: KEY

It is rare to find learned men who are clean, do not stink and have a sense of humour.

Duchess of Orléans

No work = no credit

Warm-ups (1 pt each): $-5^2 = \underline{-25}$ $(-4)^2 = \underline{16}$ $\frac{0}{3} = \underline{0}$

1.) (1 pt) The quote above is attributed to the Duchess of Orléans regarding Gottfried Leibniz (the co-discover/inventor of calculus). What special qualities did Leibniz have? Use complete English sentences.

He bathed regularly.

2.) (16 pts) Find the derivatives of the following: (Simplification is optional).

a.) (4 pts) $y = 6 + 3x^7 - \frac{4}{x^5} + \sqrt[3]{x} - \frac{1}{x}$

$$y' = 21x^6 + 20x^{-6} + \frac{1}{9}x^{-8/9} + x^{-2}$$

b.) (4 pts) $y(x) = (4x^3 - x^5 + 7x) \cdot (2x - 6x^8)$

$$y'(x) = (12x^2 - 5x^4 + 7)(2x - 6x^8) + (2 - 48x^7)(4x^3 - x^5 + 7x)$$

c.) (4 pts) $z = \frac{5x^2 - 6x}{x^4 + 7x^3}$

$$\frac{dz}{dx} = \frac{(10x - 6)(x^4 + 7x^3) - (4x^3 + 21x^2)(5x^2 - 6)}{(x^4 + 7x^3)^2}$$

d.) (4 pts) $f(x) = 5\sqrt{3x^7 - 2x}$

$$\frac{5}{2} (3x^7 - 2x)^{-1/2} (21x^6 - 2)$$

3.) (4 pts) Find $f''(x)$ if $f(x) = \sqrt{x} - 3x^5$.

$$f'(x) = \frac{1}{2}x^{-1/2} - 15x^4$$

$$f''(x) = -\frac{1}{4}x^{-3/2} - 60x^3$$

4.) (4 pts) If $f(x) = \left(\frac{2x-x^3}{4x^5-6x}\right)^8$, find $\frac{df}{dx}$

$$f'(x) = 8 \left(\frac{2x-x^3}{4x^5-6x}\right)^7 \cdot \frac{(2-3x^2)(4x^5-6) - (20x^4-6)(2x-x^3)}{(4x^5-6x)^2}$$

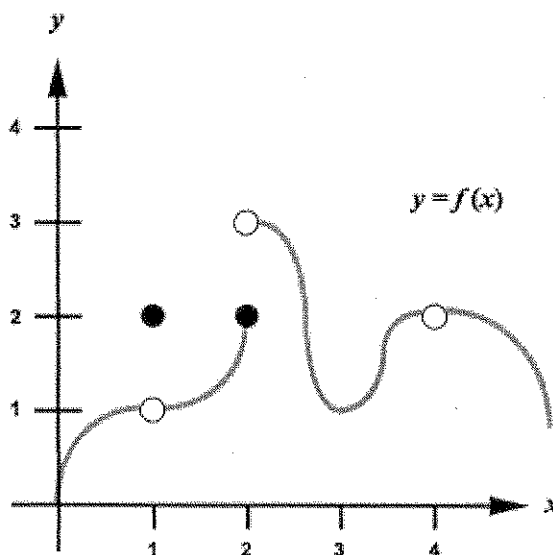
5.) (5 pts) Given the function $f(x)$ shown in the graph to the right, evaluate the following:

a.) $f(1) = 2$

b.) $\lim_{x \rightarrow 2^+} f(x) = 3$

c.) $\lim_{x \rightarrow 2^-} f(x) = 2$

d.) $\lim_{x \rightarrow 2} f(x) = \text{DNE}$



e.) Is $f(x)$ continuous at $x=1$? Explain why or why not using the definition of continuity?

NO since $\lim_{x \rightarrow 1} f(x) \neq f(1)$

6.) (4 pts) Consider $f(x) = \begin{cases} 3x-2, & x < 4 \\ x+10, & x > 4 \end{cases}$

a.) $f(4)$ is undefined

b.) $\lim_{x \rightarrow 4^+} f(x) = \underline{14}$

c.) $\lim_{x \rightarrow 4^-} f(x) = \underline{10}$

d.) $\lim_{x \rightarrow 4} f(x) = \underline{DNE}$

7.) (4 pts) Consider the function $g(x) = x^2$.

a.) Find $g(4.2) = 17.64$

b.) Find the equation of the tangent line when $x = 4$

Slope: $g'(x) = 2x \Rightarrow g'(4) = 8$

pt: $(4, 16)$

Tangent line: $y - 16 = 8(x - 4) \Rightarrow y = 8x - 16$

c.) Find the y value on the tangent line from (b.) corresponding to $x = 4.2$.

$y = 8(4.2) - 16 = 17.6$

d.) Compare the results of (a.) and (c.) and explain the similarity of the results.

The tangent line approximates g near $x=4$

8.) (4 pts) Consider the function: $g(x) = \frac{3x^2 - 6x}{5x^3 - 4x}$

1pt ... work
3pts ... no lim,
2pts ... 0,0 work.

a.) $\lim_{x \rightarrow -\infty} g(x) = \underline{0}$

b.) $\lim_{x \rightarrow +\infty} g(x) = \underline{0}$

$\lim_{x \rightarrow -\infty} \frac{\frac{3}{x} - \frac{6}{x^2}}{5 - \frac{4}{x^2}} = \frac{0}{5} = 0$

9.) (4 pts) Use the definition of the derivative to find $f'(x)$ if $f(x) = x^2 - 3x + 5$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - 3(x+h) + 5 - (x^2 - 3x + 5)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - x^2 + 3x - 5}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \\
 &= \lim_{h \rightarrow 0} (2x + h - 3)
 \end{aligned}$$

$$f'(x) = 2x - 3$$

10.) (7 pts) Below are the total cost and revenue functions for producing and selling x units.

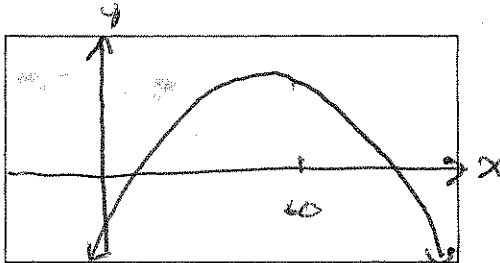
$$C(x) = 3600 + 100x + 2x^2$$

$$R(x) = 500x - 2x^2$$

a.) Write the total profit function $P(x)$.

$$\begin{aligned}
 P(x) &= (500x - 2x^2) - (3600 + 100x + 2x^2) = -4x^2 + 400x - 3600 \\
 &= -4x^2 + 400x - 3600
 \end{aligned}$$

b.) Graph the $P(x)$ on the window $[-20, 100, -5000, 8000]$ and draw what you see in the box below.



c.) Find and interpret $P(60)$. = 6000

A profit of \$6000 is earned when ~~the~~ both items are sold.

d.) Find and interpret $P'(60)$.

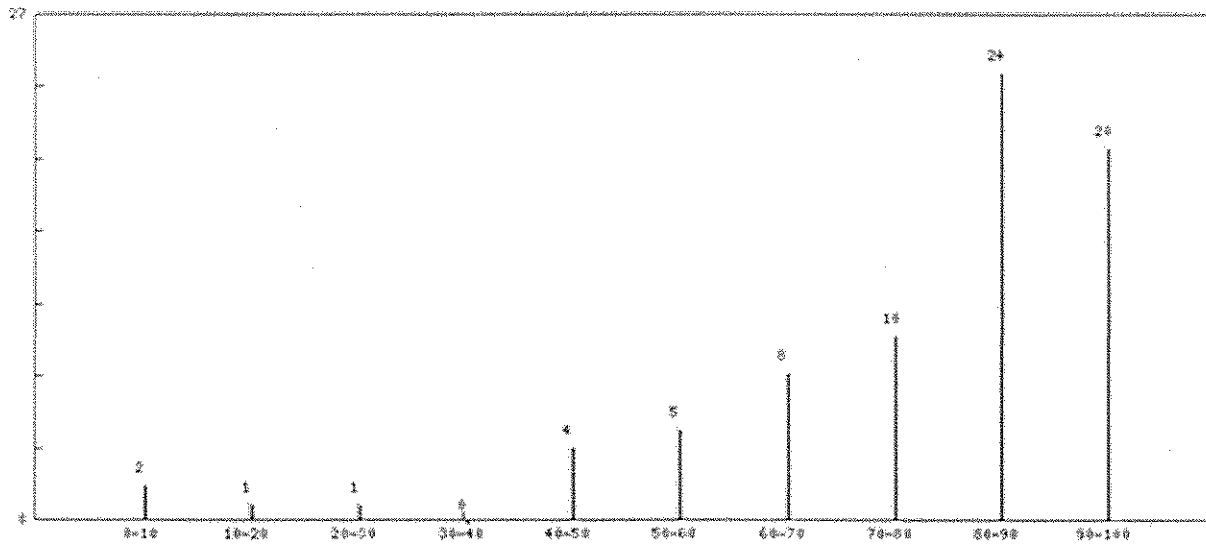
$$MP(60) = -80$$

The company loses about \$80 on the sale of the both unit.

e.) Find and interpret $P(60) - P(59)$. = 76

The company loses exactly \$76 on the sale of the both unit.

Math 148: Test 1 results?



Mean = 78.6%

Median = 84.2%