$\boxed{\begin{matrix}10.3a \\ 1/2\end{matrix}}$

Arithmetic Sequences.

Find a_n if $a_1 = -1$, $a_2 = 1$, $a_3 = 3$, $a_4 = 5, \ldots$

$a_n = 2n - 3$.

More generally, if the 1st term is a_1 & each term is d larger than the previous.

$$a_n = a_1 + (n-1)d$$

What happens when you add terms in an arithmetic sequence?

$$S_{500} = \sum_{k=1}^{500} (2k-3) = -1 + 1 + 3 + \ldots + 997$$

More generally

$$S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} a_1 + (k-1)d.$$

(recall that $a_n = a_1 + (n-1)d$).

10.3a
2/2

Geometric Sequences

Find a_n if $a_1 = 2$, $a_2 = 6$, $a_3 = 18$, $a_4 = 54$, ...

$a_n = 2 \cdot 3^{n-1}$

more generally, if the 1st term in a geometric sequence is a_1 & it grows by a factor of r @ each step, $a_n = a_1 r^{n-1}$

What happens when we sum a geometric sequence?

$$S_{100} = \sum_{k=1}^{100} 2 \cdot 3^{k-1}$$

more generally

$$S_n = \sum_{k=1}^{n} a_1 r^{k-1}$$

Ex: Choice of \$10,000/day for 2 mo. or 1 penny on one square of a chess board that doubles each day

10.3b
7/4

Recall

$$S_{500} = \sum_{k=1}^{500} 2 \cdot 3^{k-1} = \frac{(2 \cdot 3^0 + 2 \cdot 3^1 + \dots + 2 \cdot 3^{k-1})(3-1)}{(3-1)}$$

$$= \frac{\cancel{2 \cdot 3^1} + \cancel{2 \cdot 3^2} + \dots + 2 \cdot 3^k - 2 \cdot 3^0 - \cancel{2 \cdot 3^1} - \dots - \cancel{2 \cdot 3^{k-1}}}{3-1}$$

$$= \frac{2 \cdot 3^k - 2 \cdot 3^0}{3-1}$$

$$= \frac{\cancel{2}(3^k - 1)}{\cancel{2}}$$

more generally

$$S_N = \sum_{k=1}^{N} a_1 r^{k-1} = \frac{(a_1 r^0 + a_1 r^1 + \dots + a_1 r^{N-2} + a_1 r^{N-1})(r-1)}{r-1}$$

$$= \frac{\cancel{a_1 r^1} + \cancel{a_1 r^2} + \dots + \cancel{a_1 r^{N-1}} + a_1 r^N - a_1 r^0 - \cancel{a_1 r^1} - \dots - \cancel{a_1 r^{N-1}}}{r-1}$$

$$= \frac{a_1 r^N - a_1 r^0}{r-1}$$

$$= \frac{a_1 (r^N - 1)}{r-1}$$

10.3
24

proof by mathematical induction.

conjecture: $S_n = \sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1(r^n-1)}{r-1}$

□ proof.

$n=1$: $S_1 = \sum_{k=1}^{1} a_1 r^{k-1} = a_1 r^0 = a_1 = \frac{a_1(r^1-1)}{(r-1)}$ True.

$n=k$: Assume $S_k = \sum_{m=1}^{k} a_1 r^{m-1} = \frac{a_1(r^k-1)}{r-1}$

$n=k+1$: Need to show that $S_{k+1} = \frac{a_1(r^{k+1}-1)}{r-1}$

so, write down the left side and show it is equivalent ~~th~~ to the right side.

$$S_{k+1} = \sum_{m=1}^{k+1} a_1 r^{m-1}$$

$$= \underbrace{a_1 r^0 + a_1 r^1 + \dots + a_1 r^{k-1}} + a_1 r^k$$

(by assumption) $= \frac{a_1(r^k-1)}{r-1} + a_1 r^k$

$a_1 r^{k+1} - a_1 r^k$

$$= \frac{\cancel{a_1 r^k} - a_1 + a_1 r^k(r-1)}{r-1}$$

$$= \frac{a_1 r^{k+1} - a_1}{r-1}$$

$$= \frac{a_1(r^{k+1}-1)}{r-1}.$$

∴ our conjecture is proved ∎.

10.3.b
3/4

What are the monthly payments
on a 30 yr conventional loan of $200,000
at 6% compounded monthly

Think. 360 separate loans $P_1, P_2, \dots, P_{360}$.
Loan P_k will be paid off in full after k months.
The balance paid off will remain constant at R.

So, R is the monthly payment.

$\boxed{\begin{matrix}10.3b \\ 4/4\end{matrix}}$

30 yrs → 360 payments of R.

0 1 2 3 … 359 360

R R R … R R

$R = \cancel{200000}\, P_1 (1+\frac{.06}{12})^1$

$R = P_2 (1+\frac{.06}{12})^2$

$R = P_3 (1+\frac{.06}{12})^3$

$\vdots$

$R = P_{359} (1+\frac{.06}{12})^{359}$

$R = P_{360} (1+\frac{.06}{12})^{360}$

$P_1 = R(1+\frac{.06}{12})^{-1}$

$P_2 = R(1+\frac{.06}{12})^{-2}$

$P_3 = R(1+\frac{.06}{12})^{-3}$

$\vdots$

$P_{359} = R(1+\frac{.06}{12})^{-359}$

$P_{360} = R(1+\frac{.06}{12})^{-360}$

I know $\sum_{k=1}^{360} P_k = 200000$

$\Rightarrow 200000 = \sum_{k=1}^{360} R\underbrace{(1+\frac{.06}{12})}_{r}^{-k}$

$\Rightarrow 200000 = R\left[r^{-1} + r^{-2} + \dots + r^{-360}\right]\frac{(r^1-1)}{(r^1-1)}$

$= \frac{R\left[r^0 + \cancel{r^{-1}} + \dots + \cancel{r^{-359}} - \cancel{r^{-1}} - \dots - \cancel{r^{-359}} - r^{-360}\right]}{r-1}$

$= \frac{R(1-r^{-360})}{r-1} \Rightarrow R = \frac{200000(\frac{.06}{12})}{(1-(1+\frac{.06}{12})^{360})}$

Review.

composition

Find $f \circ g$ and $g \circ f$ and their domains if

1) $f(x) = \sqrt{x-16}$ and $g(x) = x^2$

2) $f(x) = \sqrt{x^2+8}$ and $g(x) = \sqrt{x^2-9}$

inverses

1) $f(x) = 4 - \sqrt{x+5}$

2) $f(x) = \dfrac{4x}{2-x}$

3) $f(x) = \dfrac{e^x - e^{-x}}{2}$.

Graph $\dfrac{-(x+4)^2}{(x+2)(x-7)}$.