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## Sequences and Series

Natural numbers  $\mathbb{N} = \{1, 2, 3, \dots\}$

$f(N) = 2N + 5$  where  $D_f = \{N | N \in \mathbb{N}\}$

$f(1)$

$f(2)$

$f(3)$

$f(N)$  is an example of a sequence

Special Notation:  $a_N = 2N + 5$ .

$N$  is the index and  $a_N$  is the  $N^{\text{th}}$  term.

Ex 1: Write the 1st 4 terms of

1)  $a_N = \frac{3N+1}{N+1}$

2)  $b_N = 3^N + (-2)^N$

3)  $c_1 = 7$ ;  $c_N = c_{N-1} - 4$  for  $N \geq 2$

Ex 2: Find the  $N^{\text{th}}$  term in the sequence...

1)  $3, 5, 7, 9, \dots$

2)  $4, 9, 16, 25, \dots$

3)  $7, -7, 7, -7, \dots$

4)  $-x, \frac{x^2}{2}, -\frac{x^3}{4}, \frac{x^4}{8}, \dots$

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In Finance ( $\Sigma$  calculus) we often want to add terms of a sequence.

Ex 3: If  $a_n = 3n - 1$ , ~~add~~ Find  $a_1 + a_2 + a_3 + a_4$

Special notation.  $\sum_{k=1}^4 (3k - 1)$

more generally  $\sum_{k=1}^n a_k$  ← dummy index.

Ex 4: Find

1)  $\sum_{k=1}^4 k$

2)  $\sum_{k=1}^4 \frac{1}{10^k}$

3)  $\sum_{k=1}^4 (-1)^k$  vs  $\sum_{k=1}^4 -1^k$

Ex 5: Write out

1)  $\sum_{k=1}^4 \frac{(-2)^{k+1}}{k}$

2)  $\sum_{k=1}^4 (-1)^{k+1} (2k-1)^2$

3)  $\sum_{k=1}^4 \frac{(-1)^{k+1}}{k} x^k$