$\boxed{\begin{matrix} 4.3a \\ 1/47 \end{matrix}}$

Log. Functions.

$$y = b^x \iff x = \log_b(y) \quad \text{for } b > 0 \text{ and } b \neq 1$$

Logs and exponentials are inverse fcts.

<u>Ex 1</u>: Write in Log Form.

a) $64 = 4^3$

b) $2 = \sqrt[3]{8}$

c) $\frac{1}{16} = 4^{-2}$

d) $10^{-3} = 0.001$

e) $e^0 = 1$

f) $x^4 = z$

remember, log & exp. form are equivalent.

<u>Ex 2</u>: Write in exponential form.

a) $\log_2(8) = 3$

b) $\log_{25}(5) = \frac{1}{2}$

c) $\log_e(e) = 1$

d) $\log_{10}(1000) = 3$

e) $\log_b(a) = c$.

4.3 4
2/3/3

What exactly are logs?

$f(x) = 2^x$ Then $f^{-1}(x) = \log_2(x)$

Exponential Function

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

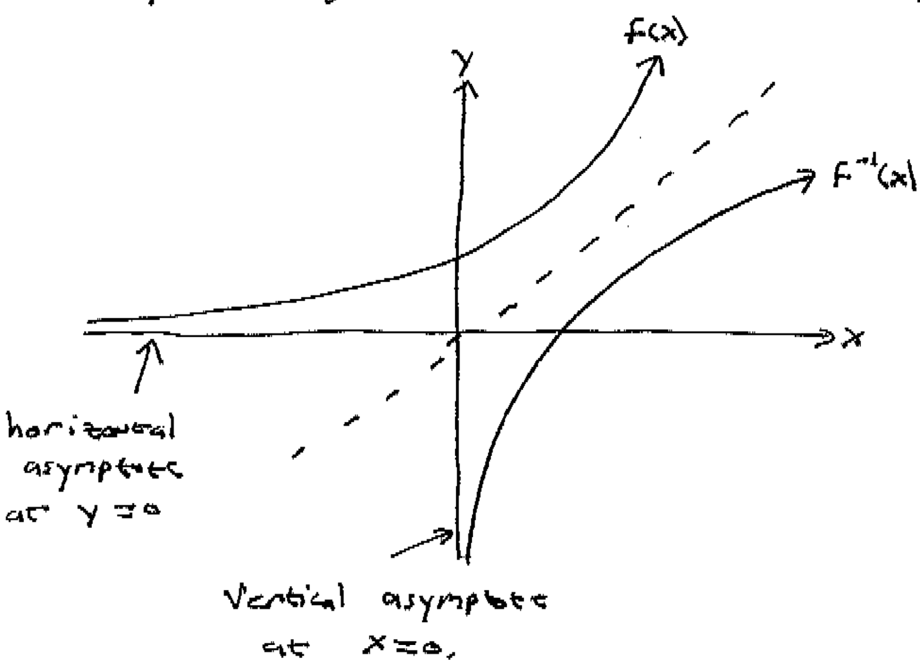
Logarithmic Function.

x	$f^{-1}(x)$
	-3
	-2
	-1
	0
	1
	2
	3

$(x, y) \longleftarrow$ becomes $\longrightarrow (y, x)$

$D_f : \{x \mid x \in \mathbb{R}\}$ $\qquad D_{f^{-1}} : \{x \mid x > 0\}$

$R_f : \{y \mid y > 0\}$ $\qquad R_{f^{-1}} : \{y \mid y \in \mathbb{R}\}$.



4.3a
3/43

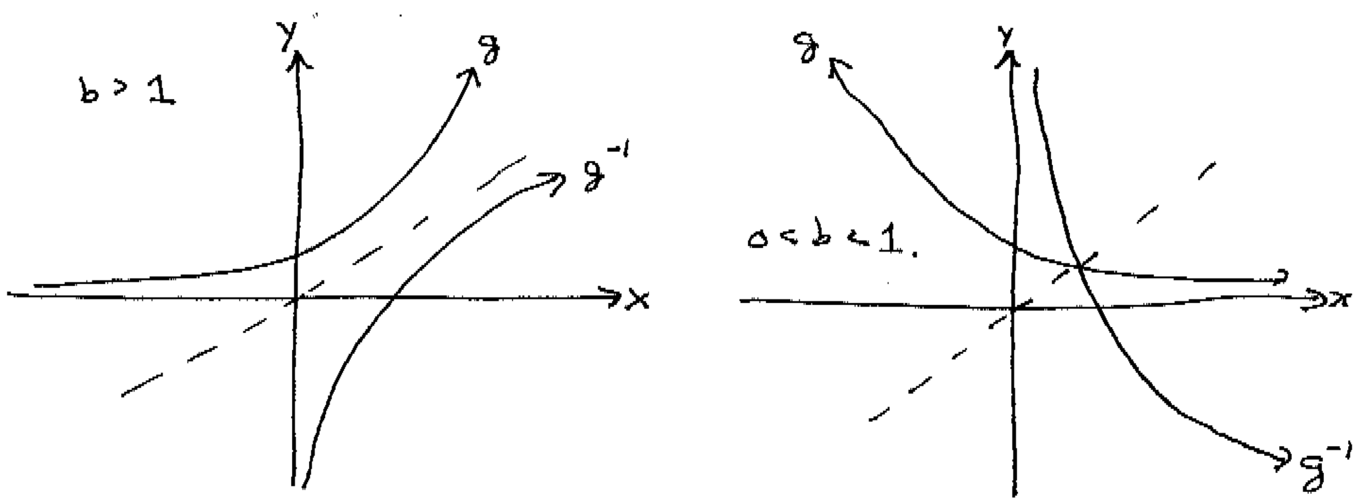
So $f(x) = 2^x$ and $f^{-1}(x) = \log_2(x)$ are inverse fcts.

$$f(f^{-1}(x)) = 2^{\log_2(x)} = x, \; x > 0$$

$$f^{-1}(f(x)) = \log_2(2^x) = x, \; x \in \mathbb{R}$$

MORE GENERALLY:

If $g(x) = b^x$ then $g^{-1}(x) = \log_b(x)$



Ex 3: $h(x) = 4^x$, find $h^{-1}(x)$.

Ex 4: $f(x) = \log_3(x+5)$, find $f^{-1}(x)$.

$$y = \log_3(x+5)$$

$$\iff 3^y = x + 5$$

$$\iff 3^y - 5 = x$$

$$\iff f^{-1}(x) = 3^x - 5.$$

4.3b
1/3

$y = b^x \iff x = \log_b(y)$ for $b > 0$ & $b \neq 1$.

Things to remember.

$\log_{10}(x) = \log(x)$ common log

$\log_e(x) = \ln(x)$ Natural log.

Ex 1: Find.

$\log_b(1)$

$\log_b(b)$

Ex 2: Solve $\log_b(x) = 3$.

Remember:

$b^{\log_b(x)} = x, \quad x > 0$

$\log_b(b^x) = x, \quad x \in \mathbb{R}$

Exponential and Log Rules

E1: $b^{x+y} = b^x b^y$ L1: $\log_b(xy) = \log_b(x) + \log_b(y)$

E2: $b^{x-y} = \dfrac{b^x}{b^y}$ L2: $\log_b\left(\dfrac{x}{y}\right) = \log_b(x) - \log_b(y)$

E3: $(b^x)^a = b^{xa}$ L3: $\log_b(x^a) = a\log_b(x)$.

4.3b
2/3

To prove the log rules.

Let $x = b^u \iff u =$ _____

$y = b^v \iff v =$ _____

Also, recall that $\log_b(b^{\boxed{}}) = \boxed{}$

L1) $\log_b(xy) = \log_b(b^u \cdot b^v)$

$= \log_b(b^{u+v})$

$= u + v$

$= \log_b(x) + \log_b(y)$

L2) $\log_b\left(\frac{x}{y}\right) = \log_b\left(\frac{b^u}{b^v}\right)$

$= \log_b(b^{u-v})$

$= \cancel{\log_b}\ u - v$

$= \log_b(x) - \log_b(y)$.

L3) $\log_b(x^a) = \log_b((b^u)^a)$

$= \log_b(b^{au})$

$= au$

$= a\log_b(x)$.

Ex 3: work w/ log rules.

a) $\log_3(6) - \log_3(2)$

b) $2\log_3(2) + \frac{1}{2}\log_3(4)$

c) $\log_4(50) - 2\log_4(5)$

d) $\frac{2}{3}\log_2(27) - 3\log_2(4)$

e) $\log_5(5\sqrt{5})$

f) $\ln\left(\frac{\sqrt{e}}{e^3}\right)$

Ex 4: If $\log_b(2) = u$ and $\log_b(3) = v$, express the following in terms of u & v.

a) $\log_b\left(\frac{3}{2}\right)$

b) $\log_b(6)$

c) $\log_b\left(\frac{16}{27}\right)$

d) $\log_b(\sqrt[3]{18})$

Ex 5: Find all integers between $\log_2(1)$ and $\log_2(128)$

Ex 6: Find the sum of all prime numbers between $\log_2(\frac{1}{2})$ and $\log_2(256)$.

Ex 7: Is $f(x) = \log(x-3) + \log(2-x)$ a function?

... DOMAIN ...

4.3c
2/2

Ex 2: Solve for x.

a) $\log(x^2 - 2x - 2) = 2\log(x-2)$

b) $\ln(x+8) - \ln(x) = 3\ln(2)$

c) $\ln(x) + \ln(x+4) = \frac{1}{2}\ln(9)$

d) $\ln(x) + \ln(x-1) = \ln(2)$

Ex 3: Find the inverse.

a) $f(x) = 5^{3x-1} + 4$

b) $g(x) = 3^{2x-3} \div 2$

c) $h(x) = 3\ln(5x-2)$

d) $f(x) = 2 + \ln(5x-3)$

Ex 4: Prove $1 = -1$.

$\ln(1) = \ln((-1)^2)$

$\Rightarrow 0 = 2\ln(-1)$

$\Rightarrow 0 = \ln(-1)$

$\Rightarrow 1 = -1$.

Ex 5: Find the domain.

a) $f(x) = \ln(x-3)$

b) $g(x) = \ln(2-x)$

c) $h(x) = \ln(x-3) + \ln(2-x)$.

$\boxed{\begin{matrix} 4.3c \\ 1/2 \end{matrix}}$

Recall: $b^0 = 1 \iff \log_b(1) = 0$

$b^1 = b \iff \log_b(b) = 1$

our friend <u>BOB</u>: $y = b^x \iff x = \log_b(y)$ for $b>0$ & $b \neq 1$

LOG RULES: for $b>0$ & $b\neq 1$, ~~$x,y>0$~~ $x,y>0$ and $a \in \mathbb{R}$

<u>L1</u> $\log_b(xy) = \log_b(x) + \log_b(y)$.

<u>L2</u> $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$.

<u>L3</u> $\log_b(x^a) = a\log_b(x)$

<u>Ex 1</u>: Write as the sum of logs of 1st. deg. polys.

a) $\log_b\left(\dfrac{x^2}{\sqrt{x+1}}\right)$

b) $\log_b(x^4 + x^3 - 20x^2)$

c) $\log_b(x^5 + 5x^4 - 14x^3)$