3.1
1/3

Polynomial Functions.

$f(x) = 3x - 4$ 1st degree poly.

$g(x) = 9x^3 + 6x^2 - 30$ 3rd degree poly.

$a_n \neq 0.$

$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$ nth degree

~~P has 1 zero on 1 at at x = n if P(n) = 0.~~

Polynomial Long Division.

Ex 1: Divide $9x^3 + 6x^2 - 30$ by $3x - 4$.

$$
\begin{array}{r}
3x^2 + 6x + 8 + \dfrac{2}{3x-4} \\
3x - 4 \,\big)\, 9x^3 + 6x^2 + 0x - 30 \\
-(9x^3 - 12x^2) \\
\hline
18x^2 + 0x \\
-(18x^2 - 24x) \\
\hline
24x - 30 \\
-(24x - 32) \\
\hline
2
\end{array}
$$

3.1
2/13

Synthetic Division.

Ex 2: Divide $9x^3 + 6x^2 - 30$ by $x - 1$

$$\begin{array}{r|rrrr} & 9 & 6 & 0 & -30 \\ & & 9 & 15 & 15 \\ \hline 1 & 9 & 15 & 15 & -15 \end{array}$$

$$= 9x^2 + 15x + 15 + \frac{-15}{x-1}$$

check: $(x-1)\left[9x^2 + 15x + 15 - \frac{15}{x-1}\right]$

Ex 3: Divide $3x^4 - 11x^3 - 18x + 8$ by $x - 4$

$$\begin{array}{r|rrrrr} & 3 & -11 & 0 & -18 & 8 \\ & & 12 & 4 & 16 & -8 \\ \hline 4 & 3 & 1 & 4 & -2 & 0 \end{array}$$

$$= 3x^3 + x^2 + 4x - 2.$$

so $\underbrace{3x^4 - 11x^3 - 18x + 8}_{P(x)} = (3x^3 + x^2 + 4x - 2)(x - 4)$

find $P(4)\ldots$

Ex 4: $\underbrace{3x^4 - 16x^2 - 3x + 7}_{P(x)} \div x + 2$ write as $x - (-2)$.

$$\begin{array}{r|rrrrr} & 3 & 0 & -16 & -3 & 7 \\ & & -6 & 12 & 8 & -10 \\ \hline -2 & 3 & -6 & -4 & 5 & -3 \end{array}$$

$$P(x) = (x - (-2))(3x^3 - 6x^2 - 4x + 5) - 3$$

$P(-2) = \ldots$

$\boxed{\begin{array}{c} 2.1 \\ 2b \end{array}}$

polys w/ odd vs. even degree.

polys w/ pos. vs. neg. leading coefficients.

If $P_n(x)$ is an n^{th} degree poly w/ real coefficients.

1) P_n is cont.

2) P_n is smooth.

3) P_n has <u>at most</u> n roots (think quadratic).

4) P_n has at most $n-1$ turning points.

<u>Ex5</u>: Plot $P(x) = x^3 - 4x^2 - 4x + 16$ $\quad -3 \le x \le 5$ w/ pt. plotting & synthetic division.