2.5a
1/

COMBINING FUNCTIONS

In order to work w/complicated functions, you need some tools for analyzing & recognizing functions.

We will learn

1) Toolkit Functions

2) Transformations

3) Operations on functions.

4) composition of functions.

1) <u>Toolkit functions</u>   (10 elementary functions).

Name & Definition	Graph	Domain & Range	Important Points Symmetry.
1. constant			
2. linear or identity			
3. absolute value.			
4. quadratic			
5. cubic.			
6. square root.			
7. cube root			
8. reciprocal			
9. sine			
10. exponential			

2) <u>Transformations</u>, part 1

    Heating Schedule Overhead.

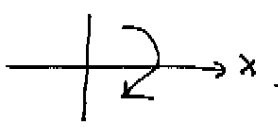   Lighthouse overhead

2.5b
1/2

Transformations   (part 2 cont.)

List   the   kinds   of   transformations.

shifts

stretches & compressions.

flips.

type	y-transformations	x-transformations
shifts	↑ ↓	→ ←
stretch & compress	⬍ ✳	⟷ ⤫
flips		

graph $f(x) = 2(x+3)^2 + 2$ w/transformations.

$$g(x) = \sqrt{x+2}$$

$$h(x) = 5\sqrt{-\tfrac{1}{4}x + \tfrac{1}{4}} - 2$$

⟹ Determine base ft.
[factor,
list transformation
graph step by step

$\boxed{\begin{array}{c} 2.56 \\ \hline 4.2 \end{array}}$

(3)   Operations w/ functions.

If $f(x) = \sqrt{1-x}$ and $g(x) = \sqrt{4+x}$

$D_f : \{x \mid x \leq 1\}$ $D_g : \{x \mid x \geq 4\}$.

$\underset{1}{\underline{\qquad + \quad \overset{\circ}{\downarrow} \quad - \qquad}}$ $\underset{-4}{\underline{\qquad - \quad \overset{\circ}{\downarrow} \quad + \qquad}}$

$(f + g)(x) = f(x) + g(x) = \sqrt{1-x} + \sqrt{4+x}$

$D_{f+g} : \{x \mid -4 \leq x \leq 1\}$.

$(f - g)(x) = f(x) - g(x) = \sqrt{1-x} + \sqrt{4+x}$

$D_{f-g} = \{x \mid -4 \leq x \leq 1\}$.

$(fg)(x) = f(x) \cdot g(x) = \sqrt{1-x}\sqrt{4+x} = \sqrt{(1-x)(4+x)}$

$D_{fg} = \{x \mid -4 \leq x \leq 1\}$.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{1-x}}{\sqrt{4+x}} = \sqrt{\dfrac{1-x}{4+x}}$
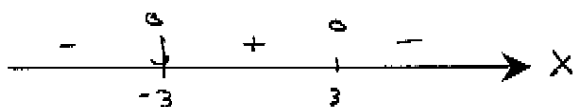
$D_{\frac{f}{g}} = \{x \mid -4 < x \leq 1\}$.
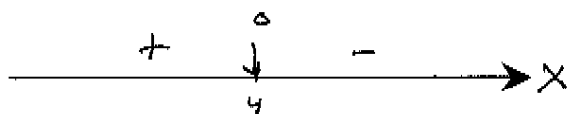
Handout – Composition
Dusty Wilson
Math 115

Example 1
Let $f(x) = \sqrt{9 - x^2}$ _____ and $g(x) = \sqrt{4 - x}$ _____.

a.) Find the domain of f.

Domain of f: $D_f = \{ x \mid$ ___$-3 \leq x \leq 3$___ $\}$

b.) Find the domain of g.

Domain of g: $D_g = $ ___$\{x \mid x \leq 4\}$___.

c.) Create a table of values.

x	-6	-5	-4	3	4	5
$f(x)$	$\sqrt{10}$	3	$\sqrt{8}$	1	0	
$g(f(x))$		0	1	$\sqrt{8}$	3	

d.) Find $(f \circ g)(x) = f(g(x))$

$$f(\sqrt{4-x}) = \sqrt{9 - (\sqrt{4-x})^2}$$
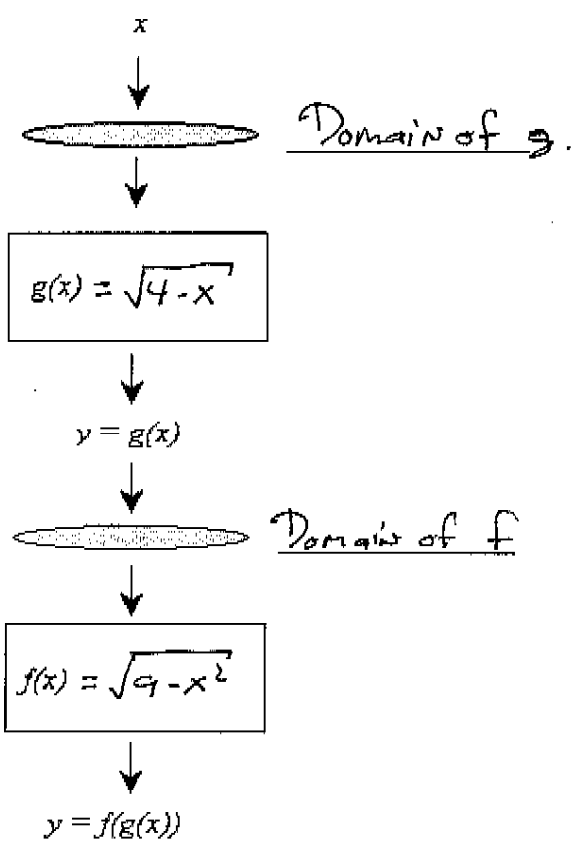$$= \sqrt{9 - 4 + x}$$
$$= \sqrt{5 + x}$$

$f(g(x)) = $ _____

e.) If we call the expression found in (d.) the apparent composition (AC), find D_{AC}.

Domain of AC: $D_{AC} = $ ___$\{x \mid x \geq -5\}$___.

f.) The domain of $f(g(x))$ is ___$D_{f \circ g} = D_g \wedge D_{AC}$___.

Domain of $f(g(x))$: $D_{f \circ g} = $ ___$\{x \mid -5 \leq x \leq 4\}$___.

x
$\downarrow$
(Domain of g.)
$\downarrow$
$g(x) = \sqrt{4-x}$
$\downarrow$
$y = g(x)$
$\downarrow$
(Domain of f)
$\downarrow$
$f(x) = \sqrt{9-x^2}$
$\downarrow$
$y = f(g(x))$

Example 2

Let $f(x) = \dfrac{x}{x+4}$ and $g(x) = 2-x$. Find $f(g(x))$ and $g(f(x))$ and the domains of each composition.

a.) Find the domain of f and g.

$$D_f = \{x \mid x \in \mathbb{R} \text{ and } x \neq -4\} \qquad\qquad D_g = \{x \mid x \in \mathbb{R}\}$$

b.) Find $f(g(x))$.

$$f(2-x) = \frac{2-x}{(2-x)+4}$$

$$= \frac{2-x}{6-x}$$

$$f(g(x)) = \frac{2-x}{6-x}$$

c.) What is the domain of the apparent composition?

$$D_{AC} = \{x \mid x \in \mathbb{R} \text{ and } x \neq 6\}.$$

d.) What is $D_{f \circ g}$?

$$D_{f \circ g} = \{x \mid x \in \mathbb{R} \text{ and } x \neq 6\}.$$

e.) Find $g(f(x))$.

$$g\left(\frac{x}{x+4}\right) = 2 - \frac{x}{x+4}$$

$$= \frac{2(x+4)-x}{x+4}$$

$$= \frac{x+8}{x+4}$$

$$g(f(x)) = \frac{x+8}{x+4}$$

f.) What is the domain of the apparent composition?

$$D_{AC} = \{x \mid x \in \mathbb{R} \text{ and } x \neq -4\}.$$

g.) What is $D_{g \circ f}$?

$$D_{g \circ f} = \{x \mid x \in \mathbb{R} \text{ and } x \neq -4\}.$$

Example 3 (2.5 #81)

Let $f(x) = \dfrac{2x+1}{x-2}$ and $g(x) = \dfrac{x+2}{x-3}$. Find $f(g(x))$ and $g(f(x))$ and the domains of each composition.

a.) Find the domain of f and g.

$\text{D}_f =$ _____ $\text{D}_g =$ _____

b.) Find $f(g(x))$.

$f(g(x)) =$ _____

c.) What is the domain of the apparent composition?

$\text{D}_{AC} =$ _____

d.) What is $D_{f \circ g}$?

$D_{f \circ g} =$ _____

e.) Find $g(f(x))$.

$g(f(x)) =$ _____

f.) What is the domain of the apparent composition?

$\text{D}_{AC} =$ _____

g.) What is $D_{g \circ f}$?

$D_{g \circ f} =$ _____