KEY

Handout – Exponentials and Logs
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Math 115

Page 1: Straight Forward.
Page 2: #7 challenging.
#8 proof.

1. Simplify: $\log_3 9 + \log_4 1 + \ln\sqrt{e}$

$\log_3(3^2) + \log_4(4^0) + \log_e(e^{1/2})$

$2 + 0 + \frac{1}{2}$

Solution: $\boxed{5/2}$

2. Write in terms of simpler logarithms: $\log_a \frac{bc}{a^2d}$

$= \log_a(bc) - \log_a(a^2d)$

$= \log_a(b) + \log_a(c) - [\log_a(a^2) + \log_a(d)]$

$\boxed{= \log_a(b) + \log_a(c) - 2 - \log_a(d)}$

Solution: ______

3. Find x: $2\log_3 x = \log_3 2 + \log_3(4-x)$

$\log_3(x^2) = \log_3[2(4-x)]$

$x^2 = 8 - 2x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$\cancel{x = -4}$ or $x = 2$

Solution: $\boxed{x = 2}$

4. Find the domain of: $f(x) = \ln(x-2) + \ln(7-x)$. Use set notation.

$x > 2$ AND $x < 7$

Solution: $\boxed{\{x \mid 2 < x < 7\}}$

5. Suppose $5000 is invested at 5.5% compounded continuously. How much is in the account after 8 years?

$P(8) = 5000e^{.055(8)}$

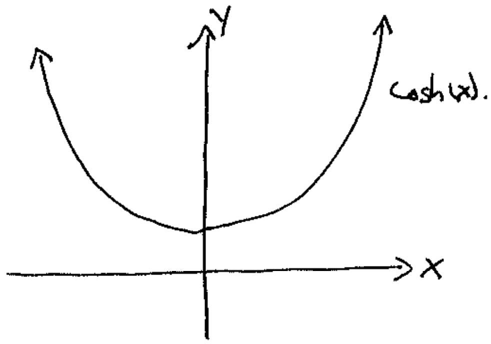
Solution: $\boxed{\$7763.54}$

6. Evaluate $\log_4 9.8765$ to 4 decimal places, and show how you got it.

$\frac{\log(9.8765)}{\log(4)}$

Solution: $\boxed{1.6520}$

7. Find the inverse of $\cosh(x)=\dfrac{e^x+e^{-x}}{2}$. Are there any restrictions on $\cosh(x)$ and $\cosh^{-1}(x)$?



cosh(x) is not 1-1.

we must restrict the domain.

$\boxed{1}$ cosh(x) on $x \geq 0$ w/range $y \geq 1$.
The inverse has domain $x \geq 1$ and range $y \geq 0$.

$\boxed{2}$ cosh(x) on $x < 0$ w/range $y \geq 1$. The inverse has domain $x \geq 1$ and range $y < 0$.

$$y = \frac{e^x + e^{-x}}{2}$$

$$x = \frac{e^y + e^{-y}}{2}$$

$$2x = e^y + \frac{1}{e^y}$$

$$2xe^y = (e^y)^2 + 1$$

$$0 = (e^y)^2 - 2xe^y + 1$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4(1)(1)}}{2(1)}$$

$$e^y = x \pm \sqrt{x^2-1}$$

$$y = \ln(x \pm \sqrt{x^2-1})$$

which do I take?

$\boxed{1}$ claim: If $x \geq 1$, then $y = \ln(x+\sqrt{x^2-1}) \geq 0$ and is consequently the inverse of cosh(x) on $x \geq 0$.

□ proof.

$* \quad x \geq 1$

$\Rightarrow x^2 \geq 1$

$\Rightarrow x^2 - 1 \geq 0$

$\Rightarrow \sqrt{x^2-1} \geq 0$

$\Rightarrow x + \sqrt{x^2-1} \geq 1$ (ADD $*$)

$\Rightarrow \ln(x+\sqrt{x^2-1}) \geq \ln(1) = 0.$ ■

$\boxed{2}$ claim: If $x \geq 1$, then $y = \ln(x-\sqrt{x^2-1})$ is negative and the inverse of cosh(x) on $x < 0$.

□ proof.

A. $x^2 - 1 < x^2$

$\Rightarrow \sqrt{x^2-1} < |x| = x$ on $x > 1$.

$\Rightarrow 0 < x - \sqrt{x^2-1}$.

B. Test $x - \sqrt{x^2-1} \overset{?}{<} 1$

$\Leftrightarrow [x - \sqrt{x^2-1}]\dfrac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}} \overset{?}{<} 1$

$\Leftrightarrow \dfrac{x^2 - (x^2-1)}{x+\sqrt{x^2-1}} = \dfrac{1}{x+\sqrt{x^2-1}} \overset{?}{<} 1$

$\Leftrightarrow x + \sqrt{x^2-1} > 1$ proved @ ☺

$\Rightarrow 0 < x - \sqrt{x^2-1} < 1$

$\Rightarrow \ln(x - \sqrt{x^2-1}) < \ln(1) = 0.$ ■

Conclusion

Function.		Inverse.
$\cosh(x),\ x \geq 0$	$\Leftrightarrow$	$\cosh^{-1}_{y \geq 0}(x) = \ln(x+\sqrt{x^2-1})$
$\cosh(x),\ x < 0$	$\Leftrightarrow$	$\cosh^{-1}_{y<0}(x) = \ln(x-\sqrt{x^2-1})$.

Solution: ______________________

NOTE: Alternately, compose inverses & show equivalence to $y = x$.

8. Prove that $\log_b(x^r) = r\log_b(x)$. What are the assumptions we must make in order for this proof to be valid?

Assume $b > 0$ and $b \neq 1$, $x > 0$, and $r \in \mathbb{R}$.

claim: $\log_b(x^r) = r\log_b(x)$.

□ proof.

Let $u = \log_b(x)$. this is true iff $b^u = x$.

now,
$$\log_b(x^r) = \log_b[(b^u)^r]$$
$$= \log_b(b^{ru})$$
$$= ru$$
$$= r\log_b(x).$$

Hence, our claim is proved. ■