100 | 90's | 80's | 70's | 60's | <60

1 | 2 | 3 | 5 | 6 | 12

Name: key.

4:06

4:18

Math 220
Spring 2024
Assessment 7
Dusty Wilson

max 100

$\bar{x} \approx 64\%$

med ≈ 61.7%

No work = no credit

You know that I write slowly. This is chiefly because I am never satisfied until I have said as much as possible in a few words, and writing briefly takes far more time than writing at length.
Carl Friedrich Gauss (1777-1855)
German mathematician

1. Warm-ups

   (a) (1 point) If $\vec{u} \perp \vec{v}$, what is $\vec{u} \cdot \vec{v}$ 0

   (b) (1 point) Real valued A has $\lambda = 7-9i$. Another λ is: $7+9i$

   (c) (1 point) If $\vec{u} \perp \vec{v}$, what is $\vec{u}^T\vec{v}$ 0

2. (1 point) Based upon the quote (above), why did Gauss write slowly? Answer using complete English sentences.

   He wanted to be concise.

3. (4 points) Find a unit vector in the direction of $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$.

$$\vec{u} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

4. (4 points) Explain how you would show that three vectors $\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ form an orthonormal set

   confirm $\|\vec{u}_1\| = \|\vec{u}_2\| = \|\vec{u}_3\| = 1$

   and $\vec{u}_1 \cdot \vec{u}_2 = \vec{u}_1 \cdot \vec{u}_3 = \vec{u}_2 \cdot \vec{u}_3 = 0$

5. (4 points) Let $\vec{y} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$. Write $\vec{y}$ as the sum of two orthogonal vectors, one in Span$\{\vec{u}\}$ and one orthogonal to $\vec{u}$.

$$\text{proj}_{\vec{u}}\vec{y} = \frac{\vec{u}\cdot\vec{y}}{\|\vec{u}\|^2}\vec{u} = \frac{-13}{26}\begin{bmatrix} 1 \\ -5 \end{bmatrix} = \begin{bmatrix} -1/2 \\ +5/2 \end{bmatrix} \in \text{span}\{\vec{u}\}$$

$$\text{and } \vec{y}^{\perp} = \begin{bmatrix} 7 \\ 4 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 5/2 \end{bmatrix} = \begin{bmatrix} 15/2 \\ 3/2 \end{bmatrix}$$

6. (2 points) True or False: If $\vec{y}$ is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without row operations. Justify your answer.

True. The weights are $c_i = \dfrac{\vec{y} \cdot \vec{u}_i}{\|\vec{u}_i\|^2}$ for each $\vec{u}_i \in$ orthogonal set

7. (2 points) True or False: An orthogonal matrix is invertible. Justify your answer.

False. It may not be square.

8. (6 points) Suppose $A = \begin{bmatrix} 19 & -15 \\ 24 & -17 \end{bmatrix}$ and $A = \rho D \rho^{-1}$ where $\rho = \begin{bmatrix} 3+i & 3-i \\ 4 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 1+6i & 0 \\ 0 & 1-6i \end{bmatrix}$.

(a) (4 points) Find a new invertible P and rotation-scaling matrix C such that $A = PCP^{-1}$.

$\lambda = 1 \pm 6i$

eigenvec.

$v = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \pm i \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$P = \begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -6 \\ 6 & 1 \end{bmatrix}$

(b) (1 point) What is the scaling factor of C?

$r = \sqrt{37}$

(c) (1 point) What is the angle (in degrees) of the rotation?

$\theta = \arctan\left(\frac{6}{1}\right) = 80.5^\circ$

9. (4 points) Find the eigenvalue(s) and corresponding eigenvectors for the matrix $A = \begin{bmatrix} 16 & -10 \\ 18 & -8 \end{bmatrix}$.

solve $0 = \begin{vmatrix} 16-\lambda & -10 \\ 18 & -8-\lambda \end{vmatrix}$

$= (16-\lambda)(-8-\lambda) + 180$

$= \lambda^2 - 8\lambda + 52$

$\Rightarrow \lambda = \dfrac{8 \pm \sqrt{64 - 4(1)(52)}}{2(1)}$

$= \dfrac{8 \pm 12i}{2}$

$= 4 \pm 6i$ eigenvalues

$4+6i$

$A - \lambda I$: $\begin{bmatrix} 16-(4+6i) & -10 \\ 18 & -8-(4+6i) \end{bmatrix}$

$\sim \begin{bmatrix} 12-6i & -10 \\ 18 & -12-6i \end{bmatrix}$ $R_1 \leftrightarrow R_2$

$\sim \begin{bmatrix} 18 & -12-6i \\ 0 & 0 \end{bmatrix}$ $\frac{1}{6}R_1 \to R_1$ $\leftarrow$ since det $= 0$

$\begin{array}{cc} 2+i & 3 \end{array}$

$\sim \begin{bmatrix} 3 & -2-i \\ 0 & 0 \end{bmatrix}$

eigen vectors: $\begin{bmatrix} 2 \pm i \\ 3 \end{bmatrix}$ or $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \pm i \begin{bmatrix} 1 \\ 0 \end{bmatrix}$