100	90's	80's	70's	60's	< 60
0	4	4	7	1	5

8:18
8:45

Name: Key

Math 220
Winter 2024
Assessment 7
Dusty Wilson

max = 98.1%
$\bar{x}$ = 73.8%
med = 76%

One time [musician] Robert Plant was set to check into the same room after I checked out, so I removed every light bulb and ordered up a bunch of stinky cheese and put it under the mattress.
Richard Marx *singer*

No work = no credit

1. Warm-ups

   (a) (1 point) If $A\vec{v} = \lambda\vec{v}$, what is $A^2\vec{v}$ $\Rightarrow \lambda^2 \vec{v}$

   (b) (1 point) Eigenvalue(s) of I_{2x2} $\Rightarrow \lambda = 1$
   mult 2

   (c) (1 point) If A is singular, then an eigenvalue is: $\lambda = 0$

2. (1 point) In reference to the quote above, what is the best practical joke you have taken part in? Answer using complete English sentences.

   I stuck a whole bunch of old railroad spikes in a friends luggage.... never heard what happened.

3. (4 points) Find the eigenspace of $A = \begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$ with associated eigenvalue $\lambda = 3$. Is every vector in this eigenspace an eigenvector?

   $$\text{rref}([A - 3I]) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

   $$E_{\lambda=3} = \text{span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

   $\vec{0} \in E_{\lambda=3}$ but is not an eigenvector.

4. (2 points) True or False: If A is diagonalizable, then A is invertible. Justify your answer.

   False. If $\lambda = 0$ is an eigenvalue, then A is singular.

5. (4 points) Find the eigenvalue(s) and eigenvectors of matrix $A = \begin{bmatrix} 6 & -2 & 0 \\ -2 & 9 & 0 \\ 5 & 8 & 3 \end{bmatrix}$.

Solve $0 = \begin{vmatrix} 6-\lambda & -2 & 0 \\ -2 & 9-\lambda & 0 \\ 5 & 8 & 3-\lambda \end{vmatrix}$

$= (3-\lambda)\left[(6-\lambda)(9-\lambda) - 4\right]$

$= (3-\lambda)(\underbrace{54 - 15\lambda + \lambda^2 - 4}_{\lambda^2 - 15\lambda + 50})$

$= (3-\lambda)(\lambda-10)(\lambda-5)$ so $\lambda = 3, 5, 10$

$\text{rref}(A-3I) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ eigenvec: $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\text{rref}(A-5I) = \begin{bmatrix} 1 & 0 & -2/9 \\ 0 & 1 & -1/9 \\ 0 & 0 & 0 \end{bmatrix}$ eigenvec: $\begin{bmatrix} 2 \\ 1 \\ 9 \end{bmatrix}$

$\text{rref}(A-10I) = \begin{bmatrix} 1 & 0 & 7/11 \\ 0 & 1 & -14/11 \\ 0 & 0 & 0 \end{bmatrix}$ w/ eigenvec $\begin{bmatrix} -7 \\ 14 \\ 11 \end{bmatrix}$

6. (4 points) The matrix $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$ has eigenvalues $\lambda = 5, 1$. Diagonalize A.

$\lambda = 5$: eigenvec(s) $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ and $\lambda = 1$: eigenvec(s) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$A = PDP^{-1}$ where $P = \begin{bmatrix} -1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

and $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. (4 points) Matrix $A = \begin{bmatrix} -2 & 12 \\ -1 & 5 \end{bmatrix}$ has eigenvectors $\vec{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. The associated eigenvalues are $\lambda = 2, 1$.

If $x_0 = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$ and $\vec{x}_t = A^t \vec{x}_0$, find a closed form expression for $\vec{x}_t$ and $\vec{x}_{equ}$

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & +4 \\ +1 & -3 \end{bmatrix}$$

$$\vec{X}_{1000} = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2^{1000} & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 8 \\ 2 \end{bmatrix}}$$

$$= \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 2^{1000} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}}$$

$$\begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

AND $\vec{X}_{equ}$ is

$$\lim_{t \to \infty} X_t = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

8. (4 points) Find the eigenvalues and a basis for each eigenspace of $\begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$

solve $0 = \begin{vmatrix} -\lambda & 1 \\ -8 & 4-\lambda \end{vmatrix}$

$$= -\lambda(4-\lambda) + 8$$

$$= \lambda^2 - 4\lambda + 8$$

$$\Rightarrow \lambda = \frac{4 \pm \sqrt{16 - 4(1)(8)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm 4i}{2}$$

$$= 2 \pm 2i$$

$\lambda = 2 + 2i$

$$A - \lambda I = \begin{bmatrix} -2-2i & 1 \\ -8 & 2-2i \end{bmatrix}$$

$$\begin{array}{cc} 2-2i & 8 \end{array}$$

$$\sim \begin{bmatrix} -8 & 2-2i \\ 0 & 0 \end{bmatrix}$$

eigenvec $\begin{bmatrix} 2-2i \\ 8 \end{bmatrix}$

AND $\lambda = 2 - 2i$ has

eigenvec $\begin{bmatrix} 2+2i \\ 8 \end{bmatrix}$