Math 220
Winter 2024
Assessment 5
Dusty Wilson

Name: key

Have you ever had a dream, Neo, that you were so sure was real? What if you were unable to wake from that dream? How would you know the difference between the dream world and the real world?
Morpheus in *The Matrix* (1999)

No work = no credit

1. Warm-ups

   (a) (1 point) Trivial solution: $\vec{0}$

   (b) (1 point) Homogeneous equation: $A\vec{x} = \vec{0}$

   (c) (1 point) A basis is: A set that is LI and spans the space.

2. (1 point) In reference to the quote above, how do you know whether you are awake or dreaming? Answer using complete English sentences.

   I believe our senses are generally reliable. If we perceive ourselves to be awake, than I believe we are awake.

3. (7 points) Prove (or disprove) the following claim.

   Claim: The set H of polynomials $p(t) = 2at + at^2$, where a is in $\mathbb{R}$, is a subspace of $\mathbb{P}_2$

proof.

Let polynomials $f(t) = 2at + at^2$ and $g(t) = 2bt + bt^2$ for $a, b \in \mathbb{R}$ and scalar k be given. $f, g \in H$.

condition 1.

If $a = 0$, then $f(t) = 0$ and so 0 is in the space.

condition 2.

$$f(t) + g(t) = 2at + at^2 + 2bt + bt^2$$
$$= 2(a+b)t + (a+b)t^2 \in H$$

condition 3.

$$k f(t) = k(2at + at^2)$$
$$= 2(ak)t + (ak)t^2 \in H$$

$\therefore$ H is a subspace.

4. (10 points) Consider matrix $C = \begin{bmatrix} 1 & 1 & 2 & -1 & 2 & 3 \\ 0 & -5 & -5 & 10 & 0 & 0 \\ 2 & 2 & 4 & -2 & 5 & 4 \\ 3 & -2 & 1 & 7 & 7 & 8 \end{bmatrix}$. Find (a.) a basis for the column space of C, (b.) the rank of C, (c.) the null space of C, (d.) a basis for row(C), and (e.) if $\vec{u} \in$ nul(C) and $\vec{v} \in$ row(C), find $\vec{v}\,\vec{u}$

(a) $C \sim \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

A basis for col$A$ = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 4 \\ 8 \end{bmatrix} \right\}$

(b) rank$(C) = 4$

(c) $x_1 = -x_3 - x_4$

$x_2 = -x_3 + 2x_4$

$x_3 = x_3$

$x_4 = x_4$

$x_5 = 0$

$x_6 = 0$

$\Rightarrow$ Nul(C) = span $\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

(d) A basis for row(C) = $\{(1,0,1,1,0,0), (0,1,1,-2,0,0), (0,0,0,0,1,0), (0,0,0,0,0,1)\}$

(e) $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$ (the spaces are orthogonal.)

5. (2 points) True or False: Col$A$ is the set of all solutions of $A\vec{x} = \vec{b}$. Justify your answer.

False. The solutions are x's while col A is the set of solutions.

bonus.

6. (4 points) Give an example of a space that is not a subspace, but satisfies at least one of the three conditions of a subspace.

The set of integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ is a space that includes 0 and is closed under addition. However, $2\pi \notin \mathbb{Z}$ so it isn't closed under scalar multiplication.

Scalar

7. (4 points) Find two matrices A and B such that $\left\{ \begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} : b, c, d \in \mathbb{R} \right\}$ is the column space of A and also the column space of B

$$b\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} -1 \\ 1 \\ 5 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ -4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b \\ c \\ d \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 5 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = B$$