Math 220
Winter 2024
Assessment 2
Dusty Wilson

Name: Key

Unfortunately, no one can be told what the Matrix is. You have to see it for yourself.

Morpheus in *The Matrix* (1999)

No work = no credit

1. Warm-ups

   (a) (1 point) $1 + 2 \times 3 =$ 7

   (b) (1 point) A square matrix: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (c) (1 point) A zero vector: $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2. (1 point) In light of the quote by Morpheus, what is an aspect of this class that must be experienced first hand? Answer using complete English sentences.

   The theory of linear isn't hard, but it's hard to explain to an outsider.

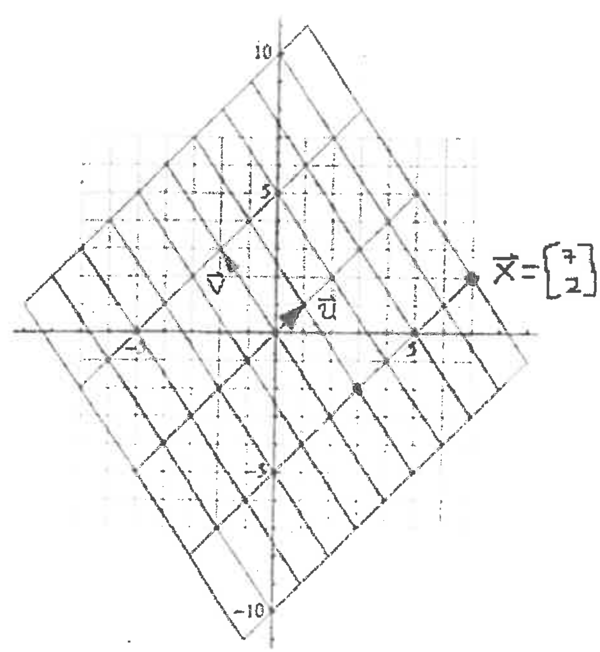
3. (4 points) Give an example of an augmented matrix in reduced echelon form that has three pivots, two free variables, and describes an inconsistent system.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 3 & 6 & \\ 0 & 1 & 0 & 2 & 5 & \\ 0 & 0 & 1 & 1 & 4 & \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

4. (4 points) (a.) Write the vector $\vec{x}$ as a linear combination of $\vec{u}$ and $\vec{v}$. And (b.) what would be the coordinates of $\vec{u} - \vec{v}$ using normal rectangular coordinates?

(a) $5\vec{u} + (-1)\vec{v}$

(b) $\vec{u} - \vec{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$



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5. (2 points) True or False: The weights $c_1, \ldots, c_p$ in a linear combination $c_1\vec{v_1} + \ldots + c_n\vec{v_n}$ cannot all be zero. Justify your answer.

false: $0\vec{v}_1 + \cdots + 0\vec{v}_p = \vec{0}$

6. (4 points) Write as a matrix equation and find two solutions.

$$x_1 \begin{bmatrix} 4 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix} \quad (1)$$

$$\Rightarrow \left[\begin{array}{cccc|c} 4 & -4 & -5 & 3 & 4 \\ -2 & 5 & 4 & 0 & 13 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & -3/4 & 5/4 & 6 \\ 0 & 1 & 1/2 & 1/2 & 5 \end{array}\right]$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 6 \\ 5 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3/4 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -9/4 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}$$

TWO SOLN: $\begin{bmatrix} 6 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ AND $\begin{bmatrix} 9 \\ 3 \\ 4 \\ 0 \end{bmatrix}$

7. (2 points) Let $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, and $\vec{v_3} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$. Does $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ span $\mathbb{R}^3$? Why or why not.

$$A = \begin{bmatrix} | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{so} \quad \vec{v}_3 = 2\vec{v}_2 - \vec{v}_1$$

$\Rightarrow$ span$\{v_1, v_2, \vec{v}_3\}$ = span$\{\vec{v}_1, \vec{v}_2\}$ and two vectors can't span $\mathbb{R}^3$, we know than $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ also can't span $\mathbb{R}^3$.

8. (4 points) Consider matrix A. Find all solutions to the homogeneous equation and write the answer in vector form (as we did repeatedly in class).

$$A = \begin{bmatrix} 0 & 1 & 5 & 7 & 4 \\ 0 & 2 & 6 & 10 & 4 \\ 0 & 3 & 7 & 13 & 4 \\ 0 & 4 & 8 & 16 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And $[A|\vec{0}] = \left[\begin{array}{ccccc|c} 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$$\Rightarrow \vec{x} = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

T2.

9. (4 points) This question is about the outline/formatting of a basic mathematical proof. Show how you would format a mathematical proof of property (vi). You do NOT need to actually prove this and may simply leave a blank space where the math/logic would normally go.

   For an extra credit point you may prove the property.

Algebraic Properties of $\mathbb{R}^n$

For all u, v, w in $\mathbb{R}^n$ and all scalars c and d:

(i) $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$

(ii) $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$

(iii) $\mathbf{u}+\mathbf{0}=\mathbf{0}+\mathbf{u}=\mathbf{u}$

(iv) $\mathbf{u}+(-\mathbf{u})=-\mathbf{u}+\mathbf{u}=\mathbf{0}$, where $-\mathbf{u}$ denotes $(-1)\mathbf{u}$

(v) $c(\mathbf{u}+\mathbf{v})=c\mathbf{u}+c\mathbf{v}$

(vi) $(c+d)\mathbf{u}=c\mathbf{u}+d\mathbf{u}$

(vii) $c(d\mathbf{u})=(cd)\mathbf{u}$

(viii) $1\mathbf{u}=\mathbf{u}$

claim: For all $\vec{u} \in \mathbb{R}^N$ and scalars c and d
$(c+d)\vec{u} = c\vec{u} + d\vec{u}$

proof,

Let $\vec{u} \in \mathbb{R}^N$ and scalars c and d be given

$$(c+d)\vec{u} = (c+d)\begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} (c+d)u_1 \\ \vdots \\ (c+d)u_N \end{bmatrix} = \begin{bmatrix} cu_1 + du_1 \\ \vdots \\ cu_N + du_N \end{bmatrix} = \begin{bmatrix} cu_1 \\ \vdots \\ cu_N \end{bmatrix} + \begin{bmatrix} du_1 \\ \vdots \\ du_N \end{bmatrix}$$

$$= c\begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} + d\begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix} = c\vec{u} + d\vec{u}$$

$\therefore (c+d)\vec{u} = c\vec{u} + d\vec{u}$. Q.E.D.

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