100	90's	80's	70's	60's	<60
0	7	10	10	3	3

7:46
7:59

Math 220
Spring 2024
Assessment 1
Dusty Wilson

high = 97.5%
$\bar{x}$ = 77.5%
med = 80%

Name: key

Life stands before me like an eternal spring with new and brilliant clothes.

Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. Warm-ups

   (a) (1 point) $-2^2 =$ -4

   (b) (1 point) Matrix with 2 pivots: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) (1 point) Matrix with 2 free variables: $\begin{bmatrix} 0 & 0 \end{bmatrix}$

2. (1 point) Gauss wrote the quote (above) when he got engaged to be married. What area do you see as an eternal spring? Answer using complete English sentences.

   I see the coming kingdom, a place w/o tears or pain, as an eternal spring.

3. (4 points) Solve the following system using the methods of this class.

$$5x_1 + 7x_2 = 11$$
$$2x_1 + 4x_2 = -4$$

$$\begin{bmatrix} 5 & 7 & 11 \\ 2 & 4 & -4 \end{bmatrix} \frac{1}{2}R_2 \to R_2 \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 21 \end{bmatrix} -\frac{1}{3}R_2 \to R_2$$

$$\sim \begin{bmatrix} 5 & 7 & 11 \\ 1 & 2 & -2 \end{bmatrix} R_1 \leftrightarrow R_2 \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7 \end{bmatrix} R_1 - 2R_2 \to R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 5 & 7 & 11 \end{bmatrix} R_2 - 5R_1 \to R_2 \sim \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & -7 \end{bmatrix}$$

$x_1 = 12$ & $x_2 = -7$

4. (2 points) What do we call a linear system that has no solutions AND give an example of such a system.

   IN CONSISTENT

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

7. (2 points) True or False: The row reduction algorithm applies only to augmented matrices for linear systems.

   Justify your answer.

False. The algorithm works on all matrices

8. (4 points) Solve the system with the given augmented matrix and write the solution in vector form.

$$\left[\begin{array}{cccc|c} 4 & 0 & 3 & 4 & 27 \\ 1 & 0 & 0 & -2 & 3 \\ 2 & 0 & 1 & 1 & 11 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 4 & 0 & 3 & 4 & 27 \\ 2 & 0 & 1 & 1 & 11 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 - 4R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 0 & 3 & 12 & 15 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 3 & 12 & 15 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 - 3R_2 \to R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad -\tfrac{1}{3}R_3 \to R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 3 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} R_2 - 5R_3 \to R_2 \\ R_1 + 2R_3 \to R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\vec{x} = \begin{bmatrix} 3 \\ 0 \\ 5 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

5. (2 points) True or False: The pivot positions in a matrix depend on whether row interchanges (swaps) are used in the row reduction process.

   Justify your answer.

False. Pivot positions are fixed throughout the process

same pivot positions

$\begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{1} \end{bmatrix} \not\sim \begin{bmatrix} \boxed{0} & 1 \\ 1 & \boxed{0} \end{bmatrix}$

6. (6 points) Solve the system with the given augmented matrix and write the solution in vector form. Circle the pivot positions in the original AND final matrix AND list the pivot columns.

$\left[\begin{array}{ccc|c} 5 & 7 & 9 & 1 \\ 3 & 5 & 7 & 9 \\ 1 & 3 & 5 & 7 \end{array}\right]$ $R_1 \leftrightarrow R_3$

$\sim \left[\begin{array}{ccc|c} \boxed{1} & 3 & 5 & 7 \\ 3 & \boxed{5} & 7 & 9 \\ 5 & 7 & 9 & \boxed{1} \end{array}\right]$ $R_2 - 3R_1 \to R_2$, $R_3 - 5R_1 \to R_3$

$\sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{array}\right]$ $-\frac{1}{4}R_2 \to R_2$

$\sim \left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -16 & -34 \end{array}\right]$ $R_3 + 8R_2 \to R_3$

$\sim \left[\begin{array}{ccc|c} \boxed{1} & 3 & 5 & 7 \\ 0 & \boxed{1} & 2 & 3 \\ 0 & 0 & 0 & \boxed{-10} \end{array}\right]$ ← NO solution