100	90's	80's	70's	60's	< 60
1	4	2	7	2	5

Name: Key

Math 220
Winter 2024
Assessment 1
Dusty Wilson

$\bar{x} = 75.9\%$

med = 76.5%

No work = no credit

This is your last chance ... You take the blue pill - the story ends, you wake up in your bed and believe whatever you want to believe. You take the red pill - you stay in Wonderland and I show you how deep the rabbit hole goes.
Morpheus in *The Matrix* (1999)

1. Warm-ups

   (a) (1 point) $-1^2 =$ -1

   (b) (1 point) Matrix with 3 rows: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   (c) (1 point) Matrix with a free variable: $\left[\begin{array}{cc|c} 1 & 2 & 3 \end{array}\right]$

2. (1 point) What is the "red pill" of academia (college)? Answer using complete English sentences.

   I think the red pill is continuing to study math and not giving up.

3. (8 points) Solve the following system using the methods of this class.

$$2x + 7y = 50$$
$$5x - 3y = 2$$

$$\left[\begin{array}{cc|c} 2 & 7 & 50 \\ 5x & -3y & 2 \end{array}\right] \; 2R_2 - 5R_1 \mapsto R_2$$

$$\left[\begin{array}{ccc} 2 & 7 & 50 \\ 0 & -41 & -246 \end{array}\right] \; -\tfrac{1}{41}R_2 \mapsto R_2$$

$$\left[\begin{array}{cc|c} 2 & 7 & 50 \\ 0 & 1 & 6 \end{array}\right] \; R_1 - 7R_2 \mapsto R_1$$

$$\left[\begin{array}{cc|c} 2 & 0 & 8 \\ 0 & 1 & 6 \end{array}\right] \; \tfrac{1}{2}R_1 \to R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 6 \end{array}\right]$$

$x = 4$ and $y = 6$.

4. (2 points) Give an example of a linear system that is inconsistent.

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 1 \end{array}\right]$$

5. (2 points) True or False: Whenever a system has free variables, the solution set contains an infinite number of solutions.

Justify your answer.

False. The system could be inconsistent.

free var

ex: $\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]$ inconsistent.

6. (8 points) Solve the system with the given augmented matrix and write the solution in vector form.

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 3 & -3 & 2 & 16 \\ 2 & -1 & 1 & 9 \end{array}\right]$$

$R_2 - 3R_1 \to R_2$

$R_3 - 2R_1 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 0 & 5 & 10 \\ 0 & 1 & 3 & 5 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 5 & 10 \end{array}\right] \quad \tfrac{1}{5}R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 + R_3 \to R_1$

$R_2 - 3R_3 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 + R_2 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

solution: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

7. (2 points) Give an example of a linear system with an infinite number of solutions

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

8. (8 points) Solve the system with the given augmented matrix and write the solution in vector form.

$$\left[\begin{array}{ccccc|c} 0 & 2 & -2 & 6 & -4 & -10 \\ 0 & 0 & 0 & 0 & 7 & 14 \\ 0 & 0 & 1 & 9 & 1 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{array}\right]$$

$\frac{1}{2}R_1 \to R_1$

$R_3 \to R_2$

$\frac{1}{7}R_2 \to R_3$

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 3 & -2 & -5 \\ 0 & 0 & 1 & 9 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{array}\right]$$

$R_1 + 2R_3 \to R_1$

$R_2 - R_3 \to R_2$

$R_4 - 2R_3 \to R_4$

$$\left[\begin{array}{ccccc|c} 0 & 1 & -1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 9 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + R_2 \to R_1$

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 12 & 0 & 0 \\ 0 & 0 & 1 & 9 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

↑ free ↑

$x_1 = x_1$

$x_2 = -12x_4$

$x_3 = 1 - 9x_4$

$x_4 = x_4$

$x_5 = 2$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -12 \\ -9 \\ 1 \\ 0 \end{bmatrix}$$