

Math 220
Spring 2024
Assessment 4c
Dusty Wilson

Name: _____

The total number of Dirichlet's publications is not large: jewels are not weighed on a grocery scale.

Carl Friedrich Gauss (1777-1855)

German mathematician

No work = no credit

1. Warm-ups

(a) (1 point) $\det(AA^{-1})$

(b) (1 point) $\vec{e}_2 \vec{e}_2^T$

(c) (1 point) $\vec{e}_2^T \vec{e}_1$

2. (1 point) Dirichlet was a mathematician around the same time as Gauss. Based upon the quote by Gauss (above), what was his view of Dirichlet's work? Answer using complete English sentences.

3. (2 points) True or false: If $\det(A) = 0$, then two rows or two columns are the same, or a row or a column is zero. Justify your answer.

4. (4 points) Match the following with the analogous term from pre-calculus:

- | | |
|-------------------|--------------------|
| (a.) null space | domain |
| (b.) column space | range |
| | codomain |
| | x -intercept(s) |
| | y -intercepts(s) |
| | the origin |

5. (4 points) What condition(s) must a space W satisfy to be a subspace? You may use words or symbols. (+1 if you are able to correctly do both).

6. (2 points) True or false: The null space of A is the solution set of the equation $A\vec{x} = \vec{0}$. Justify your answer.

7. (4 points) Prove the following:

Claim: The null space of an $m \times n$ matrix A is a subspace of \mathbb{R}^n .

8. (4 points) Define $T : \mathbb{P}^2 \rightarrow \mathbb{R}^2$ by $T(p) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix}$.

(a) (2 points) Find a non-trivial polynomial in the kernel.

(b) (2 points) Find a non-zero vector in the range.

9. (4 points) Let W be the set of all vectors of the form $\begin{bmatrix} -a+1 \\ a-6b \\ 2b+a \end{bmatrix}$ where a and b represent arbitrary real numbers. Find a set S of vectors that spans W or given an example to show that W is not a subspace.