Math 220
Spring 2024
Assessment 3
Dusty Wilson

Name: ______________________________

When a philosopher says something that is true then it is trivial. When he says something that is not trivial then it is false.
Carl Friedrich Gauss (1777-1855)
German mathematician

No work = no credit

1. Warm-ups
   (a) (1 point) I^2
   (b) (1 point) $\vec{e}_1\,\vec{e}_2^{\,T}$
   (c) (1 point) $\vec{e}_1^{\,T}\vec{e}_1$
2. (1 point) Based upon the quote by Gauss (above), what do you think Gauss' view of philosophers was? Answer using complete English sentences.
3. (8 points) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}$, find the following:
   (a) (2 points) $2A + B$
   (b) (2 points) $A^T + B$
   (c) (2 points) AB
   (d) (2 points) BA

4. (7 points) For the matrix $A_{n\times n}$, there are at least 13 statements equivalent to, "A is invertible." List at least seven of them. List more for extra credit (2 points max).

i.) A is invertible	vi.)
ii.)	vii.)
iii.)	viii.)
iv.)	xi.) (1 pt extra credit)
v.)	x.) (1 pt extra credit)

5. (4 points) Given the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ and vector $\vec{b} = \begin{bmatrix} 4 \\ 7 \\ 11 \end{bmatrix}$, solve the matrix equation $A\vec{x} = \vec{b}$ using the matrix inverse. You may use a calculator, but show enough work so that it is clear that you could do this by hand if necessary.

6. (4 points) Explain the process for finding the inverse of an $n \times n$ matrix A.

7. (2 points) True or False. If the equation $A_{n\times n}\vec{x} = \vec{0}$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix. Justify your answer.

8. (4 points) Prove the following (without reference to the invertible matrix theorem).

   Claim: If A is an invertible $n \times n$ matrix, then for each $\vec{b}$ in $\mathbb{R}^n$, the equation $A\vec{x} = \vec{b}$ has the unique solution $\vec{x} = A^{-1}\vec{b}$.