Math 220
Winter 2024
Assessment 3
Dusty Wilson

Name: ______________________________

[S]ooner or later you're going to realize just as I did that there's a difference between knowing the path and walking the path.
Morpheus in *The Matrix* (1999)

No work = no credit

1. Warm-ups

   (a) (1 point) $\vec{e}_1$

   (b) (1 point) $\vec{e}_2\,\vec{e}_1^{\,T}$

   (c) (1 point) $\vec{e}_1^{\,T}\vec{e}_2$

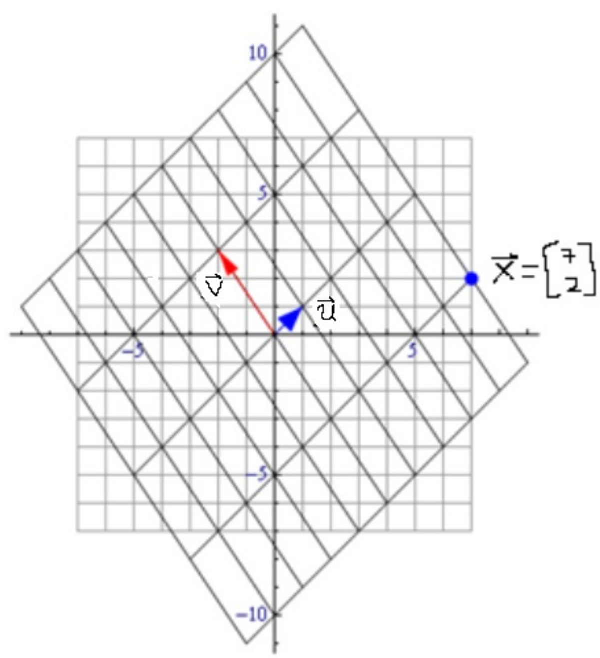
2. (1 point) In light of the quote by Morpheus, what is a path that you know and yet struggle to walk? Answer using complete English sentences.

3. (4 points) What does it mean for a set to be linearly independent?

4. (4 points) (a.) Suppose T is a linear transformation such that $T(\vec{e}_1) = \vec{u}$ and $T(\vec{e}_2) = \vec{v}$.

   (a.) Find the matrix A of the linear transformation where $T(\vec{x}) = A\vec{x}$

   (b.) find $T(\begin{bmatrix} 5 \\ -1 \end{bmatrix})$.



5. (2 points) True or False: If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S. Justify your answer.

6. (4 points) Determine if the columns of the matrix form a linearly independent set. Justify your answer.

$$A = \begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}$$

7. (4 points) Consider the linear transformation T where $T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

   a.) Describe the "image" of T geometrically.

   b.) Is T "onto"? Why or why not?

8. (4 points) Prove the following.

   Claim: If a set contains more vectors than there are entries in each vector, then the set is linearly dependent. That is, any set $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_p\}$ in $\mathbb{R}^n$ is linearly dependent if $p > n$.

9. (4 points) (a.) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^2$ that projects objects in 3D onto the $x_1\, x_2$-plane and rotates them $60°$ counter-clockwise. For example $T(\vec{e}_1) = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$ and $T(\vec{e}_3) = \vec{0}$.

   Hint: What is $T(\vec{e}_2)$

   a.) What is the matrix of the linear transformation?

   b.) Is T "one-to-one"? Why or why not?