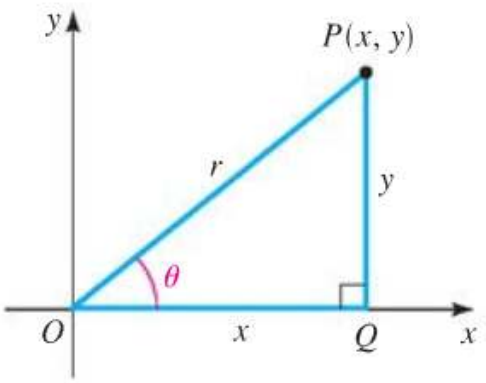
6.3 - Trig functions of Angles

Note Title 1/8/2010

This section is basically a continuation of what I've been showing you all along.

Using "right triangle trig" we can show.



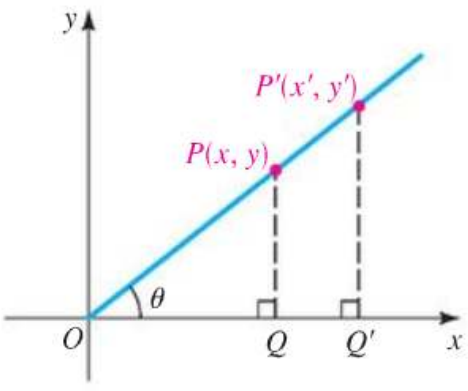
Definition of the Trigonometric Functions

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side. If $r = \sqrt{x^2 + y^2}$ is the distance from the origin to the point $P(x, y)$, then

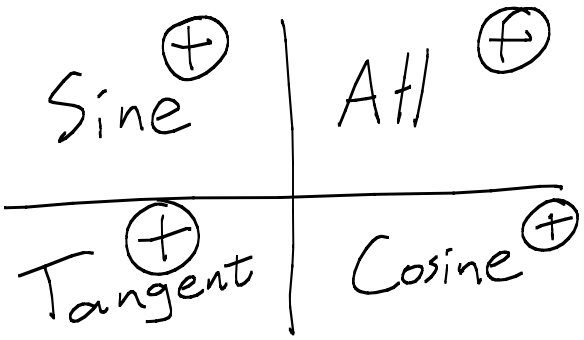
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \quad (x \neq 0)$$

$$\csc\theta = \frac{r}{y} \quad (y \neq 0) \qquad \sec\theta = \frac{r}{x} \quad (x \neq 0) \qquad \cot\theta = \frac{x}{y} \quad (y \neq 0)$$

It does not matter what point P(x, y) we choose on the angle, because the similar triangles give us the same trig ratios.



And again, a reminder of the "signs" in the different quadrants.



Signs of the Trigonometric Functions

Quadrant	Positive Functions	Negative functions
I	all	none
II	sin, csc	cos, sec, tan, cot
III	tan, cot	sin, csc, cos, sec
IV	cos, sec	sin, csc, tan, cot

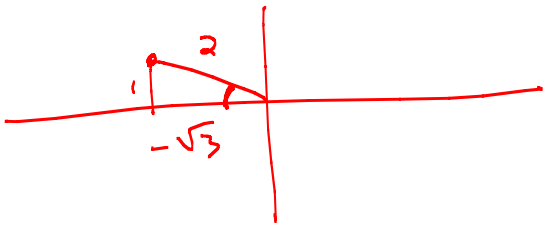
We really just need to remember

$\cos \Longleftrightarrow x$ and $\sin \Longleftrightarrow y$

and it all follows from there.

① Find

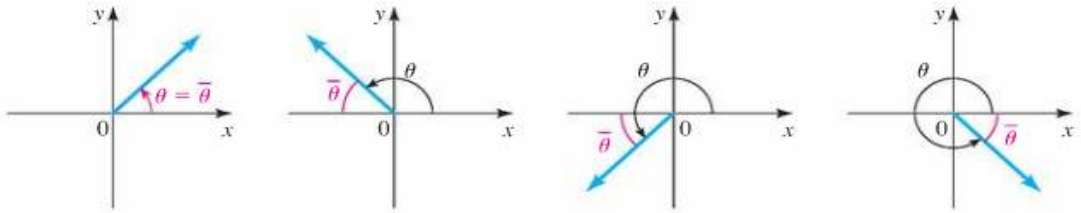
a) $\cos 150° = \frac{-\sqrt{3}}{2}$



b) $\tan 420° = \frac{\sqrt{3}}{1}$

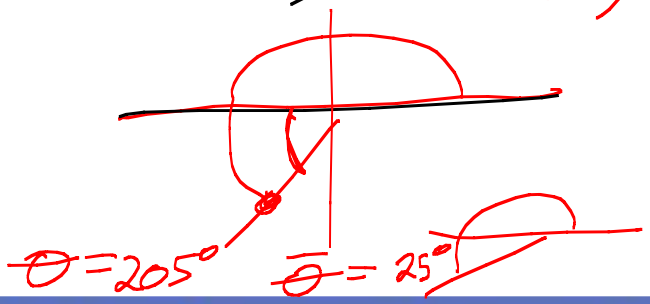
Reference Angle

Let θ be an angle in standard position. The **reference angle** $\bar{\theta}$ associated with θ is the acute angle formed by the terminal side of θ and the x-axis.



② Find the reference angle.

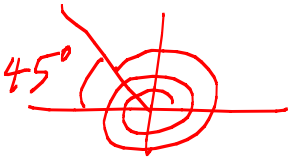
a) $\theta = \frac{4\pi}{3}$ $\bar{\theta} = \frac{\pi}{3}$



b) $\theta = 855°$

$-2 \times 360's = 720°$

$135°$



$\bar{\theta} = 45°$

Evaluating Trigonometric Functions for Any Angle

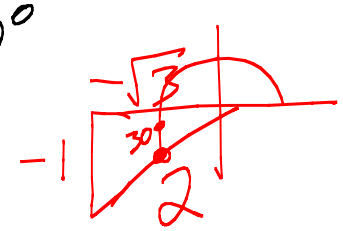
To find the values of the trigonometric functions for any angle θ, we carry out the following steps.

1. Find the reference angle $\bar{\theta}$ associated with the angle θ.
2. Determine the sign of the trigonometric function of θ by noting the quadrant in which θ lies.
3. The value of the trigonometric function of θ is the same, except possibly for sign, as the value of the trigonometric function of $\bar{\theta}$.

Or use "30°-60°-90° triangles & "Aaron's Method"

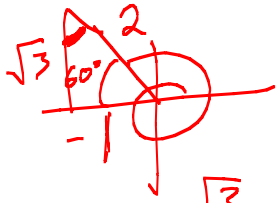
③ Find the Following

a) $\cos 210°$



$= \frac{-\sqrt{3}}{2}$

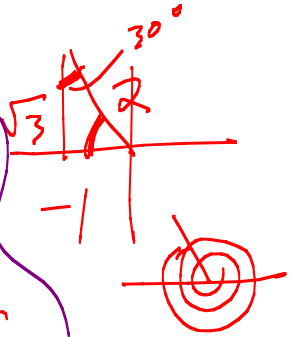
b) $\csc 480° = \frac{2}{\sqrt{3}}$



$\sin 480° = \frac{\sqrt{3}}{2}$

$\frac{2\sqrt{3}}{3}$

c) $\tan\left(-\frac{16\pi}{3}\right)$

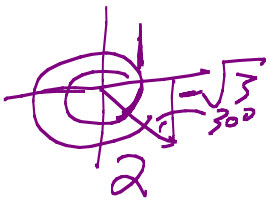


$2\pi = \frac{6\pi}{3}$

$-\frac{16\pi}{3} \Rightarrow -\frac{4\pi}{3}$

$= -\sqrt{3}$

d) $\sec \frac{11\pi}{3}$



$= \frac{r}{x} = \frac{2}{1} = 2$

$\cos\theta = \frac{1}{2} \Rightarrow \sec\frac{11\pi}{3} = 2$

And again...

Fundamental Identities

co's don't go

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

(4) Express $\cos\theta$ in terms of $\sin\theta$.

$\sin^2\theta + \cos^2\theta = 1$

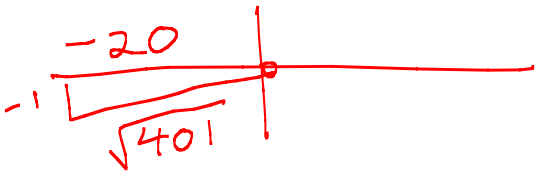
$-\sin^2\theta \qquad -\sin^2\theta$

$\sqrt{\cos^2\theta} = \sqrt{1-\sin^2\theta}$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

(5) If $\tan\theta = 1/20$ in Quad III, find $\sin\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{20} = \frac{2}{40} = \frac{100}{2000}$



$\sin\theta = \frac{-1}{\sqrt{401}}$

$a^2+b^2=c^2$

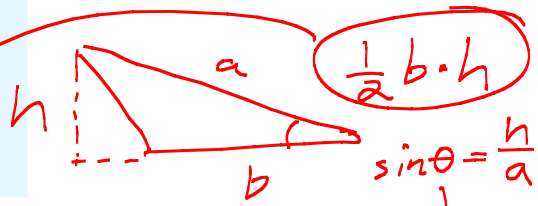
$20^2+1^2=c^2$

$\sqrt{400+1}=c$

Area of a Triangle

The area $\mathcal{A}$ of a triangle with sides of lengths a and b and with included angle θ is

$$\mathcal{A} = \tfrac{1}{2}ab\sin\theta$$



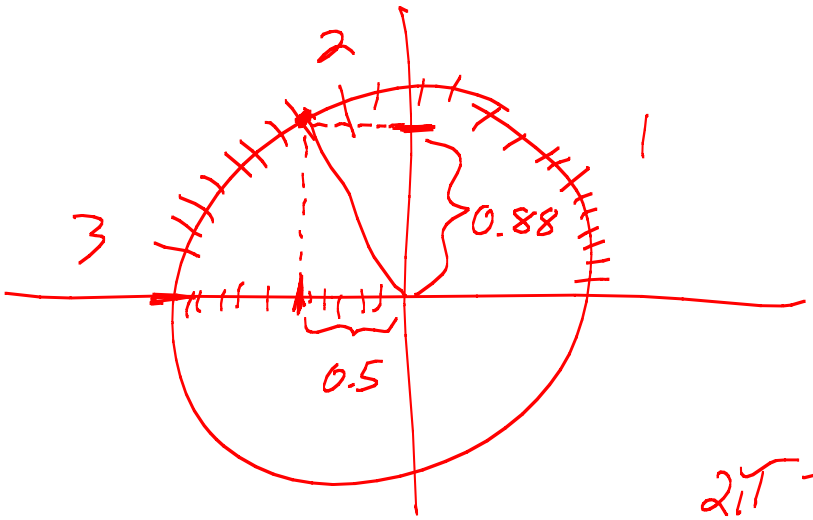
(6) Find the area of a triangle with sides length 10 & 22 and included angle 10°.

$h = a\sin\theta$

$A = \frac{1}{2}\cdot 10\cdot 22\cdot\sin 10°$

$\approx 5\cdot 22\sin 10° \approx 110\sin 10° \approx$

$\approx$ 19.1 sq. units



$2\pi - 4.2 = 2.08$

$\tan -4.2 = \frac{0.88}{-0.5} =$

6.28