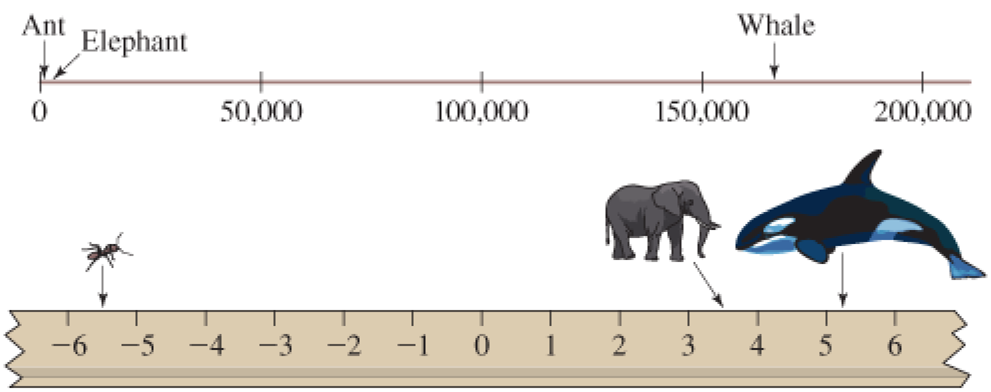
4.7 – Logarithmic Scales

When physical quantities have a very large variance, it can be useful to take the logarithm first, so that the numbers become more manageable. This is called a logarithmic scale – where numbers are are represented by their logarithms.

For example, if we needed to discuss the weights of an ant, elephant, and whale:

Animal	W (kg)	$\log W$
Ant	0.000003	−5.5
Elephant	4000	3.6
Whale	170,000	5.2



The pH Scale

$$\boxed{\mathrm{pH} = -\log[\mathrm{H}^+]}$$

Where $\left[\mathrm{H}^+\right]$ **is the concentration of hydrogen ions measures in moles per liter (M).**

If $\left[\mathrm{H}^+\right] = 10^{-3} M$ **then pH =**

Solutions with a pH of 7 are considered __________________,
those with pH<7 are ____________________ and those with pH>7 are ________________.

1. The hydrogen ion concentrations in cheeses range from 4.0×10^{-7} M to 1.6×10^{-5} M. Find the corresponding range of pH readings.

2. The most acidic rainfall ever measured occurred in Scotland in 1974; its pH was 2.4. Find the hydrogen ion concentration.

The Richter Scale: In 1935 the American geologist Charles Richter (1900–1984) defined the magnitude M of an earthquake to be

$$\boxed{M = \log \frac{I}{S}}$$

Where I is the intensity of the earthquake (measured by the amplitude of a seismograph reading taken 100 km from the epicenter) and S is the intensity of a "standard" earthquake (whose amplitude is 1 micron = 10^{-4} cm)

So the magnitude of a standard earthquake is M =

From 1900-1950 the greatest earthquake had a magnitude of 8.9, and the smallest 0. That is 800,000,000 times more intense, so Richter's scale provides more manageable #s.

3. a) Find the magnitude of an earthquake that has an intensity that is 31.25 (that is, the amplitude of the seismograph reading is 31.25 cm).

b) An earthquake was measured to have a magnitude of on the Richter scale. Find the intensity of the earthquake.

4. The 1906 earthquake in San Francisco had an estimated magnitude of 8.3 on the Richter scale. In the same year a powerful earthquake occurred on the Colombia-Ecuador border that was four times as intense. What was the magnitude of the Colombia-Ecuador earthquake on the Richter scale?

5. The 1989 Loma Prieta earthquake that shook San Francisco had a magnitude of 7.1 on the Richter scale. How many times more intense was the 1906 earthquake (see Example 4) than the 1989 event?

The Decibel Scale

$$B = 10 \log \frac{I}{I_0}$$

$I_0 = 10^{-12}\ \text{W/m}^2$ **represents the threshold of hearing, were sound is barely audible.**

So the decibel ~~left~~ of the barely audible reference sound is

$$B = 10 \cdot \log \frac{I_0}{I_0} = 10 \cdot \log 1 = 0$$

6. The intensity of the sound of traffic at a busy intersection was measured at $2{\times}10^{-5}\ \text{W/m}^2$. Find the decibel level.

I

$10^{-5} \cdot 10^{12} = 10^{7}$

$$B = 10 \cdot \log \frac{2\times10^{-5}}{10^{-12}} = 10 \log(2 \cdot 10^{7})$$

$$= 10(\log 2 + 7) \approx \boxed{73\,dB}$$

$.00002$

$$(\log 2 \cdot 10^{7}) = \log 2 + \log 10^{7}$$

$$= \log 2 + 7 \log 10 =$$

7. The decibel level of the sound from a certain hair dryer is measured at 70 dB. Find the intensity of the sound.

Source of sound	B (dB)
Jet takeoff	140
Jackhammer	130
Rock concert	120
Subway	100
Heavy traffic	80
Ordinary traffic	70
Normal conversation	50
Whisper	30
Rustling leaves	10–20
Threshold of hearing	0

$$\frac{70}{10} = \frac{10 \log \frac{I}{10^{-12}}}{10}$$

$$7 = \log \frac{I}{10^{-12}}$$

$$10^{-12} \cdot 10^{7} = \frac{I}{10^{-12}} \cdot 10^{-12}$$

$$\boxed{10^{-5} = I}$$

$.00001$

94 dB

$$\frac{94}{10} = \log\left(\frac{I}{10^{-12}}\right) \Longrightarrow 10^{9.4} = \frac{I}{10^{-12}}$$

$$10^{9.4} \cdot 10^{-12} = I = 10^{-2.6} = \boxed{2.5 \times 10^{-3}}$$

2.78×10^{-5}

3.469×10^{6}

.0000278

3,469,000