2.7 – Combining Functions

Math 141
Warnock - Class Notes

Algebra of Functions

Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows.

$$(f+g)(x) = f(x)+g(x) \qquad \text{Domain } A\cap B$$
$$(f-g)(x) = f(x)-g(x) \qquad \text{Domain } A\cap B$$
$$(fg)(x) = f(x)g(x) \qquad \text{Domain } A\cap B$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \text{Domain } \{x \in A\cap B \mid g(x)\neq 0\}$$

Domain: $\{x \neq 3\}$ $\quad [0,\infty)$

1. Let $f(x)=\dfrac{1}{x-3}$ **and** $g(x)=\sqrt{x}$ **, find the following and their domains**

a) $f+g = \dfrac{1}{x-3}+\sqrt{x}$

$[0,3)\cup(3,\infty)$

$(f+g)(4)= \dfrac{1}{4-3}+\sqrt{4}$
$= 1+2 = \boxed{3}$

b) $f-g = \boxed{\dfrac{1}{x-3}-\sqrt{x}}$

$\boxed{[0,3)\cup(3,\infty)}$

$(f-g)(4)= \dfrac{1}{4-3}-\sqrt{4}$
$= \dfrac{1}{1}-2 = \boxed{-1}$

c) $fg = \boxed{\dfrac{\sqrt{x}}{x-3}}$

$\boxed{[0,3)\cup(3,\infty)}$

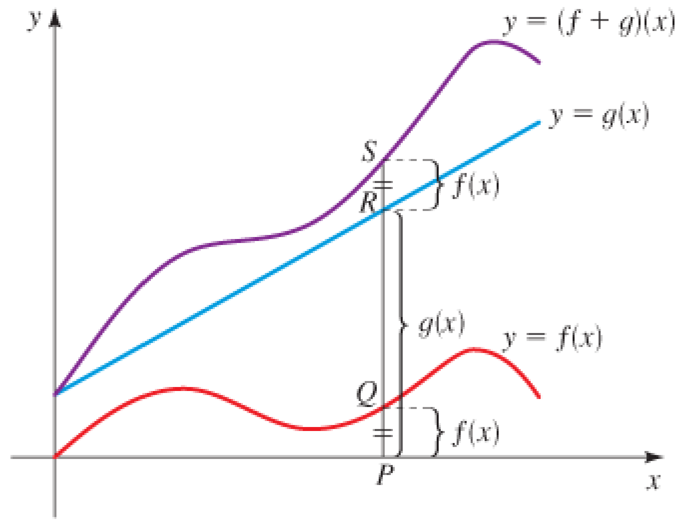
$(fg)(4)= \dfrac{\sqrt{4}}{4-3} = \boxed{2}$

d) $\dfrac{f}{g} = \dfrac{\frac{1}{x-3}}{\sqrt{x}} = \boxed{\dfrac{1}{(x-3)\sqrt{x}}}$

$\boxed{(0,3)\cup(3,\infty)}$

$\dfrac{f}{g}(4)= \dfrac{1}{(4-3)\sqrt{4}} = \boxed{\dfrac{1}{2}}$

Graphical addition

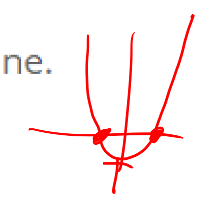


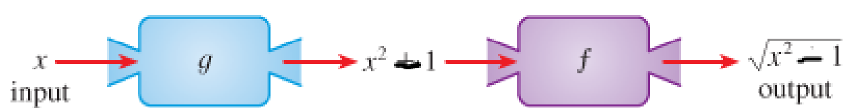
Composition of Functions

Suppose we have functions $f(x)=\sqrt{x}$ and $g(x)=x^2-1$, we can define

$$h(x)=f\left(g(x)\right)=f\left(x^2-1\right)=\sqrt{x^2-1}$$

The $\boldsymbol{h}$ machine is composed of the $\boldsymbol{g}$ machine (first) and then the $\boldsymbol{f}$ machine.





Domain g: $(-\infty,\infty)$

Range g: $[-1,\infty)$ $[0,\infty)$

Composition of Functions

Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

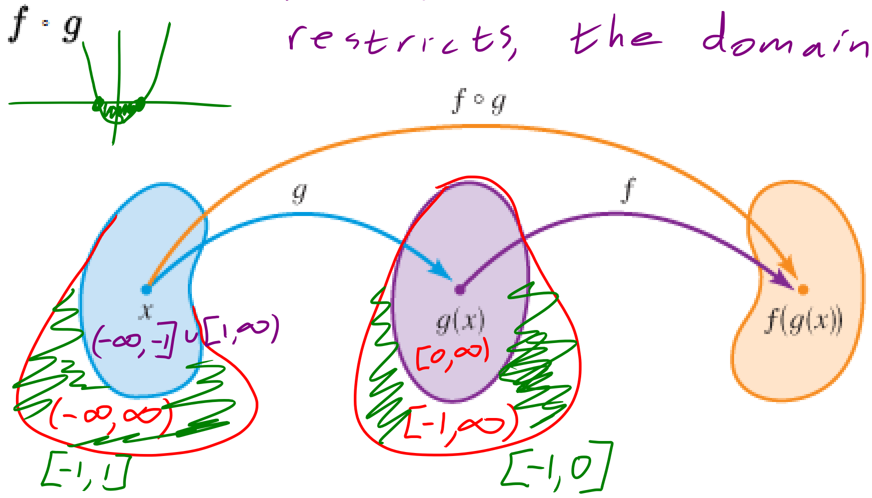
$$(f \circ g)(x) = f(g(x))$$

Domain_F: $[0,\infty)$

Domain: $f(g(x))$
$(-\infty,-1] \cup [1,\infty)$

Range of g is restricted by the Domain of f. Which in turn restricts, the domain of g.

Arrow diagram for $f \circ g$



$g(x)=x^2-1$

$f(x)=\sqrt{x}$

$\sqrt{x^2-1}$

$(x-1)(x+1)\geq 0$

-1

2. Let $f(x)=x^2$ **and** $g(x)=3-x$**, find the compositions.**

a) $f \circ g = f(g(x)) = f(3-x) = \boxed{(3-x)^2}$

All domains here $(-\infty,\infty)$

b) $g \circ f = g(f(x)) = g(x^2) = \boxed{3-x^2}$

c) $(g \circ f)(4) = 3-4^2 = 3-16 = \boxed{-13}$

d) $(f \circ g)(3) = (3-3)^2 = 0^2 = \boxed{0}$

D: $[0,\infty)$ D: $(-\infty,3]$

3. **Let** $f(x)=\sqrt{x}$ **and** $g(x)=\sqrt{3-x}$**, find the compositions and their domains.**

a) $f\circ g = f(g(x)) = f(\sqrt{3-x}) = \sqrt{\sqrt{3-x}} = ((3-x)^{1/2})^{1/2}$

$= \boxed{\sqrt[4]{3-x}}$

$\boxed{(-\infty,3]}$

$0\le \sqrt{x} \le 3$

$0 \le x \le 9$

b) $g\circ f = g(f(x)) = g(\sqrt{x})$

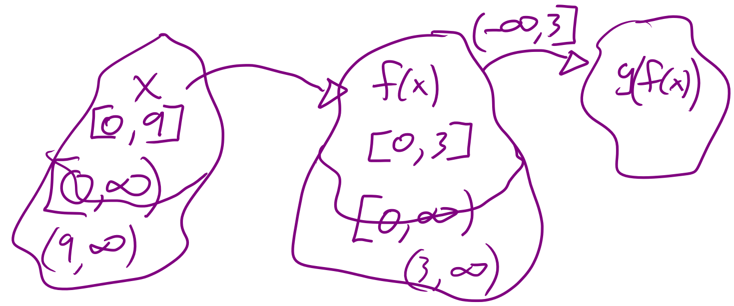
$= \sqrt{3-\sqrt{x}}$

D: $[0,\infty)$

Restriction

R: $[0,3]$

R: $[0,\sqrt{3}]$ D $\boxed{[0,9]}$



c) $f\circ f$

$= \sqrt{\sqrt{x}}$

$= \boxed{\sqrt[4]{x}}$ $\boxed{[0,\infty)}$

d) $g\circ g = g(\sqrt{3-x}) = \sqrt{3-\sqrt{3-x}}$ $(-\infty,3]$

$0 \le \sqrt{3-x} \le 3$

$0 \le 3-x \le 9$

$-3 \le -x \le +6$

$3 \ge x \ge -6$

$[0,\infty)$ coming out

acceptable: $(-\infty,3]$

range inside g $= [0,3]$

$[-6,3]$

4. **Let** $f(x)=\dfrac{1}{x}$, $g(x)=x^3$**, and** $h(x)=x^2+2$ **find** $f\circ g\circ h$.

$f(g(h(x))) = f(g(x^2+2))$

$= f((x^2+2)^3)$

$= \boxed{\dfrac{1}{(x^2+2)^3}}$

Domain: $\mathbb{R}$

$(-\infty,\infty)$

5. You have a $60 coupon from the manufacturer good for the purchase of a cell phone. The store where you are purchasing your cell phone is offering a 40% discount on all cell phones. Let *x* represent the regular price of the cell phone.

(a) Suppose only the 40% discount applies. Find a function *f* that models the purchase price of the cell phone as a function of the regular price *x*.

$\cancel{f(x) = 0.4x}$ $\quad f(x) = x - 0.4x$ ✓

$\boxed{f(x) = 0.6x}$

(b) Suppose only the $60 coupon applies. Find a function *g* that models the purchase price of the cell phone as a function of the sticker price *x*.

$g(x) = x - 60$

(c) If you can use the coupon and the discount, then the purchase price is either $f \circ g$ or $g \circ f$, depending on the order in which they are applied to the price.

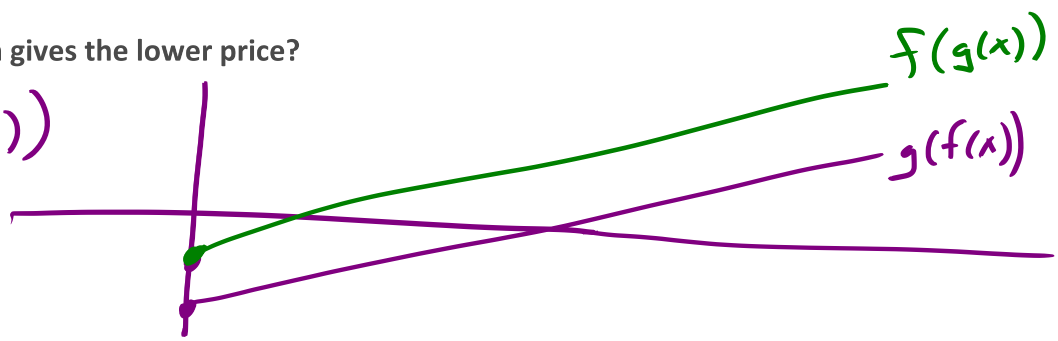
Find both $f \circ g$ and $g \circ f$.

$f(g(x)) = f(x-60) = 0.6(x-60) = .6x - 36$

$g(f(x)) = g(.6x) = .6x - 60$ ✓

(d) Which composition gives the lower price?

$g(f(x))$



credit cards
20%

Payday
390%