# 5.3 – Present Value of an Annuity & Amortization

**Math 111**

*Warnock - Class Notes*

**The <u>Present Value of an Annuity</u> is a little different than what you might think. It is the amount that would need to be deposited in one lump sum today (at the same compounded interest rate) to produce the same balance as regular payments to an annuity.**

**The formula is gnarly (what did you expect!) and you can find the derivation on page one of 5.3. We'll just use the TVM Solver to work these problems.**

**Present Value of an Ordinary Annuity**

The present value $P$ of an annuity of $n$ payments of $R$ dollars each at the end of consecutive interest periods with interest compounded at a rate of interest $i$ per period is

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] \quad \text{or} \quad P = Ra_{\overline{n}|i}.$$

**#1.** John Cross and Wendy Mears are both graduates of the Brisbane Institute of Technology (BIT). They both agree to contribute to the endowment fund of BIT. John says that he will give \$500 at the end of each year for 9 years. Wendy prefers to give a lump sum today. What lump sum can she give that will equal the present value of John's annual gifts, if the endowment fund earns 7.5% compounded annually?

**#2. A car costs ________________. After a down payment of _________________ we finance the rest at 6% interest over 36 equal monthly payments. Find the amount of the payments.**

**The interest on the 1st payment would be?**

**A loan is ______________________________ if the principal and interest are paid by a sequence of equal payments. Here is an "Amortization" formula, but we'll continue to use the TVM Solver.**

### Amortization Payments

A loan of $P$ dollars at interest rate $i$ per period may be amortized in $n$ equal periodic payments of $R$ dollars made at the end of each period, where

$$R = \frac{P}{\left[\dfrac{1-(1+i)^{-n}}{i}\right]} = \frac{Pi}{1-(1+i)^{-n}} \quad \text{or} \quad R = \frac{P}{a_{\overline{n}|i}}.$$

**#3. The Martinez family buys a house for _____________________ with a down payment of __________________. They take a 30-year mortgage at an annual interest rate of 4%.**

**a) Find the amount of the monthly payment needed to amortize this loan.**

**b) Find the amount of interest paid when the loan is amortized over 30 years.**

**c) Find the part of the 1st three payments that is interest, and the part that is applied to reducing the debt.**

| Payment # | Payment Amount | Interest Portion | Portion to Principal | Remaining Principal |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

**#4.** **a) After 15 years (half of the loan time) how much is still left in the Principal of the home loan for the Martinez family from question #3?**

| N= | N= |
|---|---|
| I%= | I%= |
| PV= | PV= |
| **PMT=** | **PMT=** |
| FV= | FV= |
| P/Y= | P/Y= |
| C/Y= | C/Y= |
| PMT: END BEGIN | PMT: END BEGIN |

**b) At 15 years, they start to pay double payments every month, how long will it take them to pay off the remainder of the loan?**

| N= | N= |
|---|---|
| I%= | I%= |
| PV= | PV= |
| **PMT=** | **PMT=** |
| FV= | FV= |
| P/Y= | P/Y= |
| C/Y= | C/Y= |
| PMT: END BEGIN | PMT: END BEGIN |

**c) How much will their final payment be?**

| N= |
|---|
| I%= |
| PV= |
| **PMT=** |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

**#5.** **Suppose the car owner from example #2 decides that she can make larger payments of __________________ instead of the owed payments of _____________.**

**How much earlier would she pay off the loan?**

| N= |
|---|
| I%= |
| PV= |
| **PMT=** |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

**How much interest would she save?**

**What would be the amount of her last payment?**

Math 111 **Finance Worksheet D**

1. **Present Value Annuity – Monthly Payment**: Megan Model borrows \$25,000 at 7.53% compounded monthly. If she wishes to pay off the loan after 15 years, how much would the monthly payment be?

| | | |
|---|---|---|
| N=<br>I%=<br>PV=<br>**PMT=**<br>FV=<br>P/Y=<br>C/Y=<br>PMT: END BEGIN | $A = PMT\left[\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$<br>$25000 = PMT\left[\frac{1-\left(1+\frac{.0753}{12}\right)^{-12(15)}}{\frac{.0753}{12}}\right]$ | **Explorations**:<br>(a) Complete the left table below. Compare the effect of decreasing **r** on the monthly payment. By how much does the monthly payment decrease if **r** goes down by 1%?<br>(b) Complete the right table below. Compare the effect of decreasing **t** on the monthly payment. |

| r | PMT (n = 12; t = 15; A = 250000) |
|---|---|
| 9% | |
| 8% | |
| 7% | |
| 6% | |
| 5% | |
| 4% | |

| t | PMT (n = 12; r = .065; A = 135000) |
|---|---|
| 10 | |
| 15 | |
| 16 | |
| 20 | |
| 25 | |
| 30 | |

2. **Managing Debt: Cost of Home Ownership**: Bob and Barb Noxious took out an \$182,300 loan at 8.5% interest for 30 years for the purchase of a new house. The loan requires monthly mortgage payments.

(a) What is the monthly payment for this mortgage?

| |
|---|
| N=<br>I%=<br>PV=<br>**PMT=**<br>FV=<br>P/Y=<br>C/Y=<br>PMT: END BEGIN |

(b) If you paid each of the 360 payments over the 30-year period, how much did you pay for the \$182,300 house over the life of the loan?

| | |
|---|---|
| ∑Int(first pmt #, last pmt # )=<br>Total Interest =<br>Total Home Cost = | **Explorations**:<br>• What original home value would lead to a total payout of \$1 million? |

Total Payout = (Monthly Payment) × (Number of payments)
Principal Paid = (Original Loan Amount) – (Present Value of Loan). At the end of 30 years, PV = 0.
Interest Paid = (Total Payout) – (Principal Paid)

(c) If you wanted to pay off the loan after having paid 10 years of payments, how much would you have to pay?

| |
|---|
| N= |
| I%= |
| **PV=** |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

(d) How much interest would have been paid over the 10 years?

| |
|---|
| ∑Int(first pmt #, last pmt #) = |

(e) How much equity would they have in the house at this time? Assume the value of the house is still $182,300

| |
|---|
| ∑Prn(first pmt #, last pmt #) = |

(f) Calculate the future value of the loan for different values of **t** (and hence, **N**). How does the future value change as *t* increases?

| t | PV (n = 12; r = .085; PMT = 1401.73) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 5 | |
| 10 | |
| 20 | |
| 25 | |
| 28 | |
| 29 | |
| 30 | |

(g) Suppose Bob and Barb bought their home 10 years ago and made monthly payments as scheduled. They plan to move in two years. They could refinance for 7.25% right now on a new 20-year mortgage, but closing costs would be $1800. Should they refinance? Assume that they will roll over the closing costs into the new mortgage.

| |
|---|
| N= |
| I%= |
| PV= |
| **PMT=** |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

| | Current Mortgage (8.5%) | New Mortgage (7.25%) |
|---|---|---|
| Present Value | | |
| Monthly Payment? | | |
| Savings per month? | | |
| Number of months to recoup the closing costs? | | |

(h) What if the refinance rate was 7.75%? Would the refinance still make sense for Bob & Barb?

| | Current Mortgage (8.5%) | New Mortgage (7.75%) |
|---|---|---|
| Present Value | | |
| Monthly Payment? | | |
| Savings per month? | | |
| Number of months to recoup the closing costs? | | |

(i) If, on the original loan, they paid an additional $100 per month, how long would it take to pay off the loan?

| | |
|---|---|
| **N=**<br>I%=<br>PV=<br>PMT=<br>FV=<br>P/Y=<br>C/Y=<br>PMT: END BEGIN | **Explorations**:<br>• By how many years would the loan term be reduced if an additional $200 was added to each payment? an additional $300?<br>• What additional payment would be required to reduce the term of the loan to 15 years?<br>• When establishing a mortgage, which is usually lower, a 15-year fixed mortgage rate or a 30-year fixed mortgage rate? |

Additional $100: t = ____________; Years reduced = ________________

Additional $200: t = ____________; Years reduced = ________________

Additional $300: t = ____________; Years reduced = ________________

If term = 15 years, Additional Payment = __________________

(j) Ask a friend or relative about their current mortgage. Write the present value, monthly payment, and interest rate of the current mortgage in the table below. Complete the rest of the table. Write a couple of sentences describing the advice you would give your friend or relative.

| | Current Mortgage (r = ___%) | New Mortgage (r = 5%) |
|---|---|---|
| Present Value | | |
| Monthly Payment? | | |
| Savings per month? | | |
| Number of months to recoup the closing costs?<br>Assume closing costs are $2000. | | |

Math 111 **Finance Worksheet E**

1. **Managing Debt: Purchasing a Car**.

(a) Suppose that you are going to finance the purchase of a new $21,000 car. There are three financing options available to you: 1.9% financing for 3 years, 3.9% financing for 4 years, or 5.9% financing for 5 years. Compare the financing costs for each of the three loans. Which would be best for you and why?

| | | |
|---|---|---|
| N= 36<br>I%= 1.9<br>PV= 21000<br>**PMT=**<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN | N= 48<br>I%= 3.9<br>PV= 21000<br>**PMT=**<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN | N= 60<br>I%= 5.9<br>PV= 21000<br>**PMT=**<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN |

| **$21,000 Car Loan** | | | |
|---|---|---|---|
| Loan Term | 3 Years (1.9%) | 4 Years (3.9%) | 5 Years (5.9%) |
| Monthly Payments | | | |
| Total Number of Payments | 36 | 48 | 60 |
| Total payout during the term | | | |
| Cost to Finance - Interest | | | |

(b) Paige is offered two options when purchasing a new $17,000 car. Option 1 offers 6.75% financing for 4 years and $2500 "cash back." Option 2 offers 4.75% financing for 5 years with no cash back. The financing requires monthly payments. Find the monthly payment for each financing option. Assume that the cash back in Option 1 will be used to reduce the amount of the original loan. If Paige's goal is to pay the minimum amount for financing over the life of the loan, which option should she choose? Explain why using specific numbers.

| | |
|---|---|
| N= 48<br>I%= 6.75<br>PV=<br>PMT=<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN | N= 60<br>I%= 4.75<br>PV=<br>PMT=<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN |

2. **Explorations**:
   (a) In problem (b), assume Option 2 offers 4.5% financing. Now which option is best?
   (b) In problem (b), how much cash back should be offered so that the total amount spent on the car at the end of the terms is equal?

3. **Managing Debt: Leasing a Car**

Which vehicle should you lease? The typical term on a lease is 3 years. To determine the cost of the lease, a residual value is used. The residual value is essentially the proportion of the vehicle's original value that the vehicle will be worth in 3 years. (That is a measure of depreciation.)

| | |
|---|---|
| N= 36<br>I%= 8<br>PV= 21011<br>PMT=<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN | N= 36<br>I%= 8<br>PV= 37695<br>PMT=<br>FV= 0<br>P/Y= 12<br>C/Y= 12<br>PMT: END BEGIN |

| | Dodge Caravan | Toyota Sienna |
|---|---|---|
| List Price | $21,011 | $37,695 |
| 8% 3-year loan pymt | $658.41 | $1,181.22 |
| Total Payments | ($658.41)(36) = $23,702.76 | ($1181.22)(36) = $42,523.92 |
| Residual | 31.8% | 60% |
| Residual Value | ($21,011)(.318) = $6,681.50 | (37,695)(.60) = $22,617 |
| Total Cost | $23,702.76 – $6,681.50 = $17,021.26 | $42,523.92 – $22,617 = $19,906.92 |
| Cost per Month | $17,021.26/36 = $472.81 | $19,906.92/36 = $552.97 |

Repeat the above calculations to determine which car has the lowest cost to own.

| | Chevy Cavalier | Toyota Camry |
|---|---|---|
| List Price | $17,510 | $29,650 |
| 8% 3-year loan pymt | | |
| Total Payments | | |
| Residual | 26.3% | 63% |
| Residual Value | | |
| Total Cost | | |
| Cost per Month | | |

4. **Explorations**:
- Compare the "cost to own" of the two cars below. Which one has the lowest "cost to own?"

| | Car 1 | Car 2 |
|---|---|---|
| List Price | $23,810 | $32,950 |
| 8% 3-year loan pymt | | |
| Total Payments | | |
| Residual | 35.7% | 57.9% |
| Residual Value | | |
| Total Cost | | |
| Cost per Month | | |

5. **Managing Debt: Paying Off a Credit Card**

Dell has advertised a Dimension E521 computer for $1149 ($1218 after tax) or $35 per month. You are in need of a new computer and this model seems to satisfy all of your needs. Suppose that you pay only the minimum due of $35 (at 19.99% APR) each month on your new computer.

(a) How long will it take you to pay off the computer? How much will you have paid on the $1218 balance when the computer is finally paid off?

| |
|---|
| **N=** |
| I%= |
| PV= |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

(b) Suppose your friend purchases the same computer and has the same beginning balance or $1218. Because your friend has bad credit, the annual interest rate is 29.99%. How long will it take your friend to pay off the computer? How much will your friend have paid on the $1218 balance when the computer is finally paid off?

| |
|---|
| **N=** |
| I%= |
| PV= |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |
| PMT: END BEGIN |

6. **Explorations**
   (a) How much would you need to invest in a sinking fund each month at 5% interest compounded monthly to accumulate the $1218 needed to purchase the computer in 2 years?

   (b) How many months would it take to accumulate the $1218 needed to purchase the computer if you invest $35 in a sinking fund earning 5% interest?