5.2 – Future Value of an Annuity

Math 111

Warnock - Class Notes

Geometric Sequences

A geometric sequence is an infinite list of number of the form

$$a, ar, ar^2, ar^3, ar^4, \ldots ar^n, \ldots$$

***a* is called the _____ _____**

So the nth term of the sequence is _____

The number *r* is called the _____ _____.

#1. **List the first 6 terms of the geometric sequence with** $a = 4$ **and** $r = -3$.

#2. **Find the 9th term of the geometric sequence 3, 15, 75, 375, …**

We are interested in finding the sum S_n **of the first n terms of a geometric sequence.**

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-1}$$

We'll start by multiplying both sides by r

$$rS_n =$$

Now we'll subtract these equations from each other.

$$rS_n =$$

$$-S_n =$$

So we have

$$rS_n - S_n =$$

$$=$$

$$S_n =$$

Sum of Terms

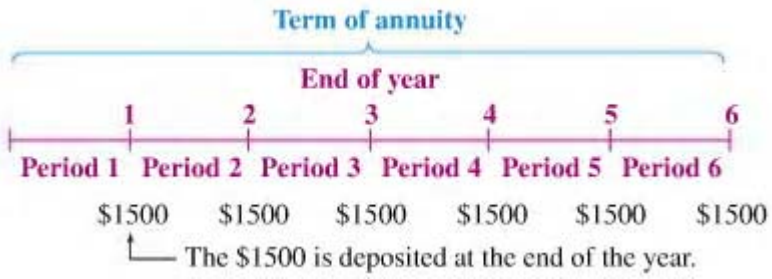
If a geometric sequence has first term a and common ratio r, then the sum S_n of the first n terms is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1.$$

#3. Find the sum of the first six terms of the geometric sequence 2, 12, 72,…

When a sequence of equal payments is made at equal periods of time, it is called an _____. Payments made at the end of each period are called _____ _____.

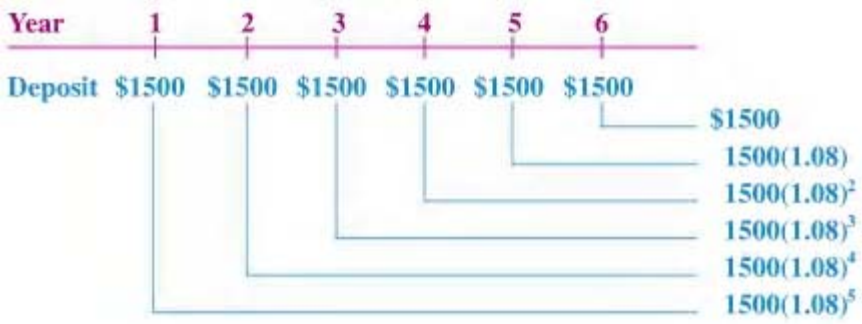
Suppose that $1500 is paid at the end of the year for 6 years, in an account paying 8% compounded annually. Each $1500 payment has to be treated separately.



How much interest will the first $1500 earn?

How much interest will the 2nd $1500 earn?

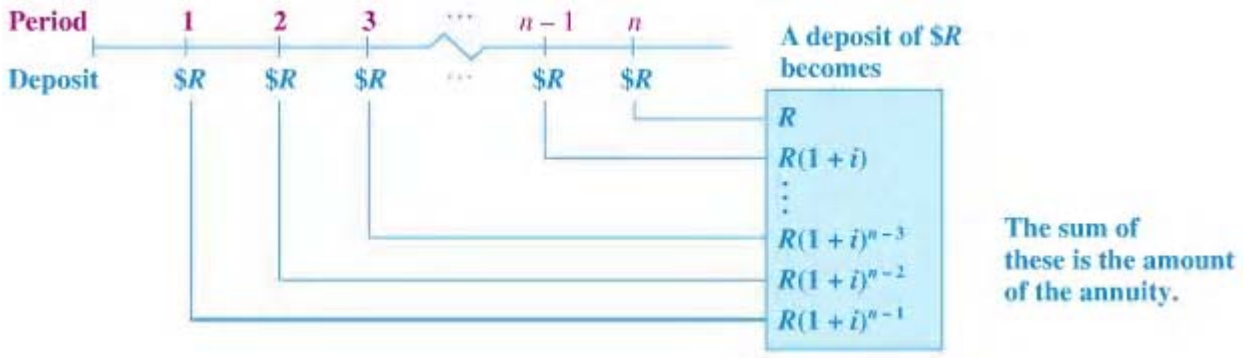
The last $1500?



What does the list of numbers on the right look like?

So we have

$\frac{a\left(r^n-1\right)}{\left(r-1\right)}$ **with a = r = n =**



So we have a geometric sequence with the first term of R and common ratio $1+i$.

$S =$

Future Value of an Ordinary Annuity

$$S = R\left[\frac{(1+i)^n - 1}{i}\right] \quad \text{or} \quad S = Rs_{\overline{n}|i}$$

where

S is the future value;

R is the periodic payment;

i is the interest rate per period;

n is the number of periods.

#4. **Hector wants to buy a used car without getting a loan. If he sets aside $200 per month in a savings account paying 3% interest compounded monthly, how much is in the account after 2 years? How much did Hector deposit? How much interest did he earn?**

#5. **Michelle is saving money for retirement. At the end of each month she puts $300 in an account that pays 8% interest compounded monthly. How much is in the account after 40 years? How much is interest and how much was deposits?**

A _____ _____ is a fund setup to receive periodic payments to produce a certain amount at some time in the future.

#6.

Experts say that the baby boom generation (Americans born between 1946 and 1960) cannot count on a company pension or Social Security to provide a comfortable retirement, as their parents did. It is recommended that they start to save early and regularly. Nancy Hart, a baby boomer, has decided to deposit $200 each month for 20 years in an account that pays interest of 7.2% compounded monthly.

a) **How much is in the account at the end of 20 years?**

b) **Nancy believe that she needs to accumulate $130,000 in the 20-year period to have enough for retirement. What interest rate would provide that amount?**

c) **Suppose Nancy can't get that interest rate, what should her monthly deposit be to have $130,000 in 20 years?**

Sinking Fund Payment

$$R = \frac{Si}{(1+i)^n - 1} \quad \text{or} \quad R = \frac{S}{s_{\overline{n}|i}}$$

where

R is the periodic payment;

S is the future value;

i is the interest rate per period;

n is the number of periods.

#7. **Buying Equipment** Harv, the owner of Harv's Meats, knows that he must buy a new deboner machine in 4 years. The machine costs $12,000. In order to accumulate enough money to pay for the machine, Harv decides to deposit a sum of money at the end of each 6 months in an account paying 6% compounded semiannually. How much should each payment be?

#8. **Savings** Beth Dahlke deposits $2435 at the beginning of each semiannual period for 8 years in an account paying 6% compounded semiannually. She then leaves that money alone, with no further deposits, for an additional 5 years. Find the final amount on deposit after the entire 13-year period.

This is actually called a "Future Value of an *Annuity Due*" – payments at the beginning of the pay-period.

$$S = R\left[\frac{(1+i)^{n+1} - 1}{i}\right] - R$$

Math 111 **Finance Worksheet B**

1. **Future Value Annuity**: How long will it take Dot Snice to accumulate $1,000,000 if she invests $3,000 per year at an annual interest rate of 8%? Assume interest is compounded annually.

N= I%= PV= PMT= FV= P/Y= C/Y= PMT: END BEGIN	$$A = PMT\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$$ $$1000000 = 3000\left[\frac{\left(1+\frac{.08}{1}\right)^{1(t)}-1}{\frac{.08}{1}}\right]$$	**Explorations**: • How long will it take to accumulate $1 million with different annual investments? • How long will it take $3000 to accumulate $1 million with different interest rates?

PMT	t (n = 1; r = .08; A = 1000000)
600	
1200	
3000	
7200	
12000	
15000	

r	t (n = 1; PMT = 3000; A = 1000000)
.01	
.05	
.08	
.09	
.13	
.20	

2. Many employers offer a 401K or 403B plan that allows employees to invest for retirement. The beauty of the plan is that employees who invest $15,000 in a year, will pay federal taxes on $15,000 less in income – a tremendous tax savings. If we assume that the tax saved equals the rate of return on an investment, calculate the return on investment for the two employees below.

Salary	$50,000	$50,000
Investment in TSA	$15,000	$0
Taxable Income	$50,000 – $15,000 = $35,000	$50,000
Fed Tax Paid	$5,308	$9,058
State Tax Paid (4%)	(.04)($35000) = $1,400	(.04)($50000) = $2,000
Tax Savings:	($9,058 + $2,000) – ($5,308 + $1,400) = $4,350	
Rate of return:	$4,350/$15,000 = 29%	

Repeat the above calculations to determine the tax savings of a second employee.

Salary	$80,000	$80,000
Investment in TSA	$15,000	$0
Taxable Income		
Fed Tax Paid	$12,902	$17,102
State Tax Paid (4%)		
Tax Savings:		
Rate of return:		