5.1 – Simple and Compound Interest

Interest on loans for less than a year is sometimes called _____ ______ ____________.
The amount borrowed is called the __________ _________.
The ________ of interest is given as a _________________ per ___________.
The ________ the money is earning interest is calculated in years.

Simple Interest

$$I = Prt$$

where
P is the principal;
r is the annual interest rate;
t is the time in years.

#1. **Marvin bought an HD TV for $2000 and financed it at 8% simple interest for 10 months. How much interest will he pay?**

If we add the principal and the interest together we have

$$P + Prt =$$

This is called the ______________ ___ _______________.

Future or Maturity Value for Simple Interest
The **future** or **maturity value** A of P dollars at a simple interest rate r for t years is

$$A = P(1 + rt).$$

#2. **Find the maturity value for the loan at simple interest.**
$12,000 loan at 5.9% for 11 months

#3. **Delinquent Taxes** An accountant for a corporation forgot to pay the firm's income tax of $321,812.85 on time. The government charged a penalty based on an annual interest rate of 13.4% for the 29 days the money was late. Find the total amount (tax and penalty) that was paid. (Use a 365-day year.)

#4. **The following is taken from** https://www.moneytreeinc.com/loans/washington/online-payday-loans

HOW MUCH DOES A PAYDAY LOAN COST?

A payday loan costs $15 per $100 borrowed up to $500, and $10 per $100 on the amount over $500. For example, if you borrowed $100 and your payday was 14 days away, the APR (Annual Percentage Rate) would be   and you would owe $115 on the date of your next payday.

Fill in the blank for your Annual Percentage Rate on this PayDay Loan.

For periods longer than a year, we use _______________ _______________.
Basically, this means that your ________________ starts earning ________________.

Let's look at $1000 at 10% per year.

Interest can also be compounded quarterly, monthly, daily.

Compound Amount

$$A = P(1 + i)^n$$

where $i = \frac{r}{m}$ and $n = mt$,

A is the future (maturity) value;
P is the principal;
r is the annual interest rate;
m is the number of compounding periods per year;
t is the number of years;
n is the number of compounding periods;
i is the interest rate per period.

#5. **Find the compound amount for**

a) $1000 at 4.5% compounded annually for 6 years.

b) $15,000 at 6% compounded monthly for 10 years.

How much interest was earned?

#6. **Comparing Investments** Two partners agree to invest equal amounts in their business. One will contribute $10,000 immediately. The other plans to contribute an equivalent amount in 3 years, when she expects to acquire a large sum of money. How much should she contribute at that time to match her partner's investment now, assuming an interest rate of 6% compounded semiannually?

It is interesting to compare loans at the same rate when simple or compound interest is used. Figure 1 shows the graphs of the simple interest and compound interest formulas with $P = 1000$ at an annual rate of 10% from 0 to 20 years. The future value after 15 years is shown for each graph. After 15 years of compound interest, \$1000 grows to \$4177.25, whereas with simple interest, it amounts to \$2500.00, a difference of \$1677.25.

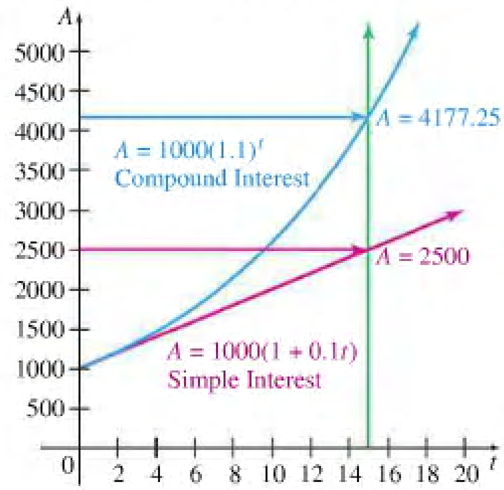

FIGURE 1

Calculate \$100 deposited at 6% compounded monthly for one year.

So the amount of interest is _____________. Which means it's actually ______ for the year.

So ______ is the ____________ or ___________ rate,

and ________ is called the _________________ rate.

In this text, to avoid confusion we will use *r* and r_E respectively.

Effective Rate

The **effective rate** corresponding to a stated rate of interest r compounded m times per year is

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1.$$

#7. **Find the effective rate corresponding to a stated rate of 5% compounded quarterly.**

#8. Joe Vetere needs to borrow money. His neighborhood bank charges 8% interest compounded semiannually. A downtown bank charges 7.9% interest compounded monthly. At which bank will Joe pay the lesser amount of interest?

The formula $A = P(1+i)^n$ has four variables, and if we know how much my we want in the future, we can determine how much needs to be saved/deposited today.

#9. If Nancy wants to have $2000 in 8 years compounded semiannually at 7%, how much should she deposit now?

Present Value for Compound Interest

The **present value** of A dollars compounded at an interest rate i per period for n periods is

$$P = \frac{A}{(1+i)^n} \quad \text{or} \quad P = A(1+i)^{-n}.$$

#10. Find the Present Value of $7500 in 9 years if money can be deposited at 5.5% compounded quarterly.

#11. **How long will it take $8,000 deposited at 3% compounded quarterly, to reach at least $23,000?** *(For this – we will use our graphing calculators.)*

#12. **Suppose the general level of inflation in the economy averages 8% per year. Find the number of years it would take for the overall level of prices to double.**

Rule of 72:

Continuous Compounding – this is where your interest is immediately earning interest. You don't have to wait monthly or daily, it happens instantaneously!

Let's look at what happens if we increase our number of compounds to more than daily. Let's use __________________ with an interest rate of _______ for _____ years.

Type of Compounding	N	Compound Amount
Quarterly	4	
Monthly	12	
Daily	360	
Every Hour		
Every Minute		

As we can see, even interest earning interest at every minute is hardly better than every hour.

(360 comes from assuming twelve 30-day months. Treasury Bills sold by the US government assume a 360-day year in calculating interest.)

Continuous Compounding

If a deposit of P dollars is invested at a rate of interest r compounded continuously for t years, the compound amount is

$$A = Pe^{rt} \text{ dollars.}$$

We will discuss the exponential function $f(x)=e^x$ more in chapter 10 when we get there, for now we'll just use it for continuous compounding.

#13. **Suppose that ________________ is deposited at 6.25% compounded continuously.**

a) **Find the compounded amount and the interest earned after 18 years.**

b) **Find the effective interest rate.**

c) **Find the time required for the original ___________ to grow to ______________.**

Summary

Simple Interest	Compound Interest	Continuous Compounding
$A = P(1 + rt)$	$A = P(1 + i)^n$	$A = Pe^{rt}$
$P = \dfrac{A}{1 + rt}$	$P = \dfrac{A}{(1 + i)^n} = A(1 + i)^{-n}$	$P = Ae^{-rt}$
	$r_E = \left(1 + \dfrac{r}{m}\right)^m - 1$	$r_E = e^r - 1$

P = principal or present value
A = future or maturity value
r = annual (stated or nominal) interest rate
t = number of years
m = number of compounding periods per year
i = interest rate per period $\quad i = r/m$
n = total number of compounding periods $\quad n = tm$
r_E = effective rate