<u>3.2 – Solving Linear Programing Problems Graphically</u>

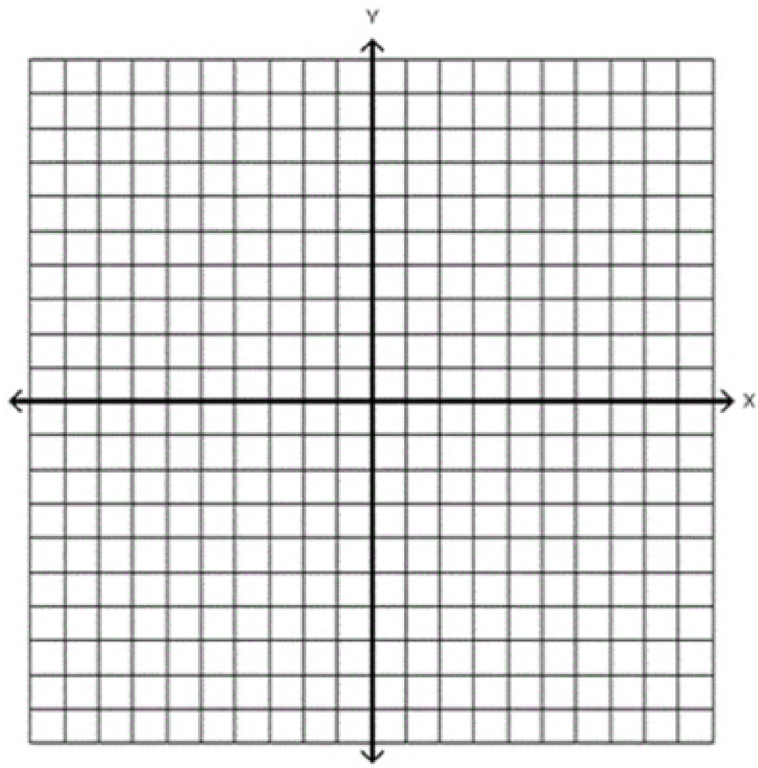
Many mathematical models in business, biology, and economics require finding an ____________________ value, either a _________________ or ________________ subject to some restrictions.

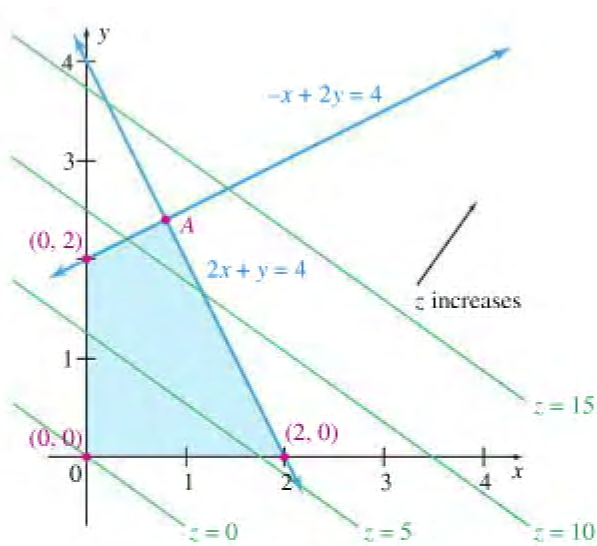
For a Linear Programming problem, there are two main components:

1. **The ____________________ ________________ - to be maximized or minimized.**
2. **The _____________________ - a set of restrictions given by inequalities.**

#1. **Find the maximum value of the objective function** $z = 3x + 4y$**, subject to the following constraints**

$$\begin{aligned} 2x + y &\le 4 \\ -x + 2y &\le 4 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



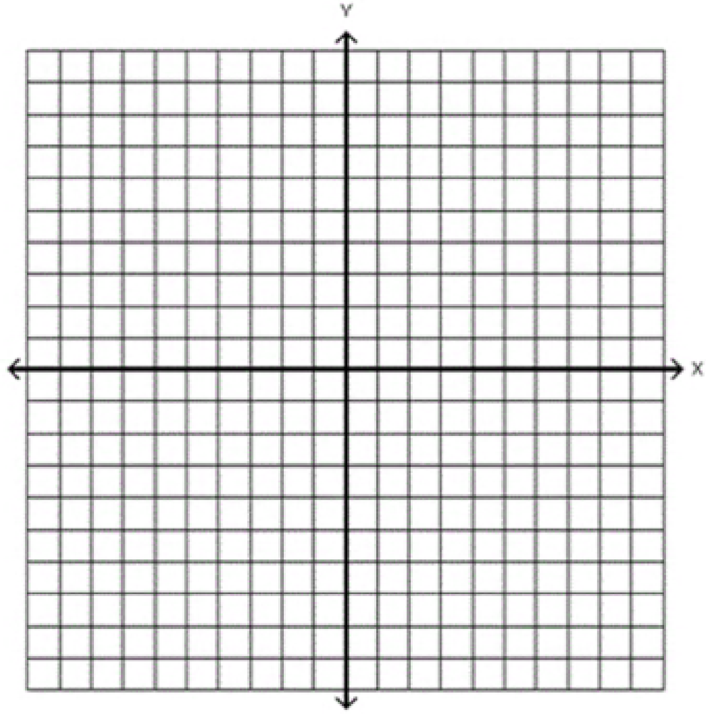


A ________________ _____________ is a point in a feasible region where two boundary lines of two constraints cross. We use solving a system of two equations to find those values.

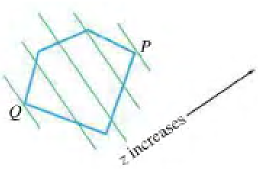
#2. Maximize $z = 4x + 5y$

subject to:

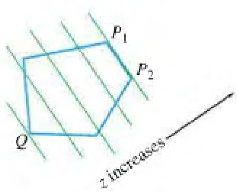
$$
\begin{aligned}
10x - 5y &\le 100 \\
20x + 10y &\ge 150 \\
x + y &\ge 12 \\
x &\ge 0 \\
y &\ge 0.
\end{aligned}
$$



There are several different cases that can occur in linear programming.



(a)



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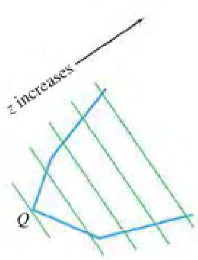

(c)

And These lead to…

Corner Point Theorem

If an optimum value (either a maximum or a minimum) of the objective function exists, it will occur at one or more of the corner points of the feasible region.

Solving a Linear Programming Problem

1. Write the objective function and all necessary constraints.
2. Graph the feasible region.
3. Identify all corner points.
4. Find the value of the objective function at each corner point.
5. For a bounded region, the solution is given by the corner point producing the optimum value of the objective function.
6. For an unbounded region, check that a solution actually exists. If it does, it will occur at a corner point.

#3. **Sketch the Feasible Region and find the maximum and minimum values of the Objective funcion** $z=4x+6y$.

$$x-y\le 3$$
$$6x-y\ge 4$$
$$3\le x+y\le 10$$