2.4 – Multiplication of Matrices

The product of a scalar *k* and a matrix *X* is the Matrix *kX*, where each element is *k* times the corresponding elements of *X*.

#1. **Calculate -2A, where**

$$A=\begin{bmatrix} 1 & 3 & -2 \\ -4 & 0 & 5 \end{bmatrix}$$

Multiplying two matrices is much more involved. Let's look at the data from last section, of food sold from convenience stores in Folsom.

	Bread	Milk	Peanut Butter	Cold Cuts
Store I	88	48	16	112
Store II	105	72	21	147
Store III	60	40	0	50

Now, say we know the prices of the items to be

- **Bread, $2 a loaf**
- **Milk, $3 a quart**
- **Peanut Butter, $4 a jar**
- **Cold Cuts, $5 a pound**

To calculate the amount of money brought in at store 1, we could set up a table like this:

	Amount		Price per Unit		Total
Bread		x		=	
Milk		x		=	
Peanut Butter		x		=	
Cold Cuts		x		=	
			(Total for Store 1)		

While this isn't overly complicated to do, it would be nice if we could do this for all the stores at once. This is where matrices come in.

We can write the "Prices" as a column matrix, $p = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$

The key for multiplying matrices, is that the _________________ of the 1st matrix must _____________ the _____________ of the 2nd matrix.

$$\begin{bmatrix} 88 & 48 & 16 & 112 \\ 105 & 72 & 21 & 147 \\ 60 & 40 & 0 & 50 \end{bmatrix} \mathbf{X} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} =$$

3 x <u>4</u> X <u>4</u> x 1 will result in a __________________

To multiply these two matrices, think about how we calculated the total from Store I on the previous page. We multiple the elements of the 1st row (matrix 1) by the elements of the 1st column (matrix 2) and add the values together.

So the product of an _______ matrix and an _________ matrix would be ________.

Product of Two Matrices

Let A be an $m \times n$ matrix and let B be an $n \times k$ matrix. To find the element in the ith row and jth column of the **product matrix** AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, and then add these products. The product matrix AB is an $m \times k$ matrix.

#2. Find the product AB of matrices

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

#3. **Given the matrices**

$$C=\begin{bmatrix} -3 & 2 \\ 0 & -1 \\ 1 & 4 \end{bmatrix} \qquad D=\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 1 \end{bmatrix}$$

a) Find the product CD.

b) Find the product DC.

c) Are they the same?

So matrix multiplication is NOT ___________________________, in other words, the order matters. AB is not the same as BA.

43. Cost Analysis The four departments of Spangler Enterprises need to order the following amounts of the same products.

	Paper	Tape	Binders	Memo Pads	Pens
Department 1	10	4	3	5	6
Department 2	7	2	2	3	8
Department 3	4	5	1	0	10
Department 4	0	3	4	5	5

The unit price (in dollars) of each product is given below for two suppliers.

	Supplier A	Supplier B
Paper	2	3
Tape	1	1
Binders	4	3
Memo Pads	3	3
Pens	1	2

a. Use matrix multiplication to get a matrix showing the comparative costs for each department for the products from the two suppliers.

b. Find the total cost over all departments to buy products from each supplier. From which supplier should the company make the purchase?