# 2.2 – Solutions of Linear Systems by the Gauss-Jordan Method

**Math 111**

*Warnock - Class Notes*

**When we deal with systems of equations that have more than two variables, we would like a more efficient way to solve them. As long as the variables are all in the same order (standard form) all we really need is the coefficients and constants.**

**We can re-write the following system**

$$\begin{aligned} x+2y-7z&=-2 \\ -2x-5y+2z&=1 \\ 3x+5y+4z&=-9 \end{aligned}$$

**In an abbreviated form like this:**

$$\begin{bmatrix} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{bmatrix}$$

**This is called a ___________________ (Plural is _____________________ )**

**Each number in the array is an ___________________ or ________________.**

**To Separate the constants in the last column from the coefficients of the variables, we put a vertical line and get an _________________ _____________.**

$$\left[\begin{array}{ccc|c} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{array}\right]$$

**The following ___________ ______________________ can be performed** *(just like the "Transformations" of systems)* **and result in an equivalent system.**

1. **Exchange any two rows**
2. **Multiply (or divide) any row by any nonzero real number**
3. **Replace any row by a nonzero multiple of that row plus a nonzero multiple of any other row.**

**#1.** **Perform the following "row operations".**

**a)** $\left[\begin{array}{ccc|c} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{array}\right] R_1 \leftrightarrow R_3 \left[\begin{array}{c} \\ \\ \\ \end{array}\right]$

**b)** $\left[\begin{array}{ccc|c} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{array}\right] \frac{-1}{2}R_2 \rightarrow R_2 \left[\begin{array}{c} \\ \\ \\ \end{array}\right]$

**c)** $\left[\begin{array}{ccc|c} 1 & 2 & -7 & -2 \\ -2 & -5 & 2 & 1 \\ 3 & 5 & 4 & -9 \end{array}\right] 2R_1 + R_2 \rightarrow R_2 \left[\begin{array}{c} \\ \\ \\ \end{array}\right]$

**The ______________ ___________________ method is an extension of the echelon method. We will first write the system as an augmented matrix. We will then use Row Operations to get the matrix into an _____________________ ______________ which is 1's down the _____________________ and 0's everywhere else.**

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & a \\ 0 & 1 & 0 & 0 & b \\ 0 & 0 & 1 & 0 & c \\ 0 & 0 & 0 & 1 & d \end{array}\right]$$

**Once we have the matrix in this form, then we can easily that**

$x = \quad , \; y = \quad , \; z = \quad , \; w =$

**#2.** **Use the Gauss-Jordan Method to solve the system of equations.**

$$\begin{aligned} 4x - 2y &= 3 \\ -2x + 3y &= 1 \end{aligned}$$

**#3.** **Use the Gauss-Jordan Method to solve the system of equations.**

$$\begin{aligned} x &= 1 - y \\ 2x &= z \\ 2z &= -2 - y \end{aligned}$$

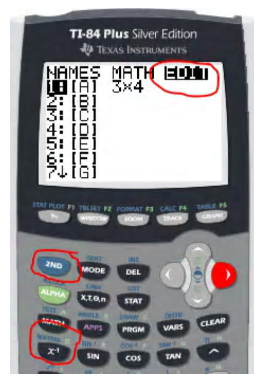
## Gauss-Jordan Method of Solving a Linear System

1. Write each equation so that variable terms are in the same order on the left side of the equal sign and constants are on the right.
2. Write the augmented matrix that corresponds to the system.
3. Use row operations to transform the first column so that all elements except the element in the first row are zero.
4. Use row operations to transform the second column so that all elements except the element in the second row are zero.
5. Use row operations to transform the third column so that all elements except the element in the third row are zero.
6. Continue in this way, when possible, until the last row is written in the form

$$[0 \quad 0 \quad 0 \quad \cdots \quad 0 \quad j \mid k],$$

   where $j$ and $k$ are constants with $j \neq 0$. When this is not possible, continue until every row has more zeros on the left than the previous row (except possibly for any rows of all zero at the bottom of the matrix), and the first nonzero entry in each row is the only nonzero entry in its column.
7. Multiply each row by the reciprocal of the nonzero element in that row.

**We can also use our calculators to solve Matrices. You will be expected to show how to solve one of these by hand on an exam, and then you can use the calculator as well on harder matrices.**




1. **First, we need to find 2nd Matrix on the calculator (above the $x^{-1}$)**
2. **Then go right to "Edit"**
3. **Then type the "size" of your matrix (row x column)**
4. **Then enter your matrix entries.**
5. **To reduce the matrix, go to Matrix → MATH and then RREF**
6. **Then enter the matrix you wish to reduce, Matrix→NAMES→ Choose your matrix**

**#4. Solve the system by Gauss-Jordan, you may use a calculator.**

$$\begin{aligned} 3x+2y-z&=-16 \\ 6x-4y+3z&=12 \\ 5x-2y+2z&=4 \end{aligned}$$

**#5.** **Solve the system by Gauss-Jordan, you may use a calculator.**

$$\begin{aligned} 3x-6y+3z&=11 \\ 2x+y-z&=2 \\ 5x-5y+2z&=6 \end{aligned}$$

**#6.** **Solve the system by Gauss-Jordan, you may use a calculator.**

$$\begin{aligned} x-z&=-3 \\ y+z&=9 \\ -2x+3y+5z&=33 \end{aligned}$$

**#7.**

**48. Manufacturing** Nadir, Inc. produces three models of television sets: deluxe, super-deluxe, and ultra. Each deluxe set requires 2 hours of electronics work, 3 hours of assembly time, and 5 hours of finishing time. Each super-deluxe requires 1, 3, and 2 hours of electronics, assembly, and finishing time, respectively. Each ultra requires 2, 2, and 6 hours of the same work, respectively.

**a.** There are 54 hours available for electronics, 72 hours available for assembly, and 148 hours available for finishing per week. How many of each model should be produced each week if all available time is to be used?

**b.** Suppose everything is the same as in part a, but a super-deluxe set requires 1, rather than 2, hours of finishing time. How many solutions are there now?

**c.** Suppose everything is the same as in part b, but the total hours available for finishing changes from 148 hours to 144 hours. Now how many solutions are there?