2.1 – Solutions of Linear Systems by the Echelon Method

Math 111

Warnock - Class Notes

In Math 91, we learned how to solve systems of equations with two equations and two unknowns, by _____. Here is an example of that.

Suppose an animal feed is made of corn and soybeans:

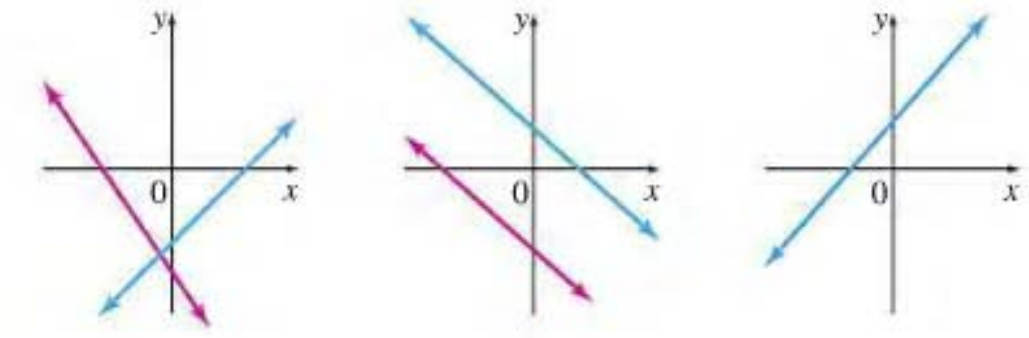
Nutritional Content of Ingredients	Corn	Soybeans
Protein	3	10
Fiber	11	4

Let's assume we want a serving of the feed to contain 115 g of protein and 95 g of fiber. How can we set up this scenario with equations?

These are _____ equations, and this is a _____ of _____ equations.

We used "elimination" to solve these in Math 91.

As a review, systems like this have three possibilities for solutions.



As we add more variables to our system, the geometric interpretation becomes more complicated, but the 3 possible results remain the same.

Because of the complexity of adding more variables, we are going to develop a more robust method for solving systems.

<u>Transformations</u> – the following 3 transformations can be applied to any system, without changing its solutions (_____ _____.)

1. **Exchange any two equations**
2. **Multiply (or divide) both sides of an equation by any nonzero real number**
3. **Replace any equation by a nonzero multiple of that equation plus a nonzero multiple of any other equation.**

The _____ _____ uses these transformations to rewrite equations of a system until it has triangular form.

$$\begin{aligned} x+ay&=b \\ y&=c \end{aligned} \qquad\qquad \begin{aligned} x+ay+bz&=c \\ y+dz&=e \\ z&=f \end{aligned}$$

Each form gives the solution to the "last variable" and then we can use _____ _____ to find the rest.

Let's show how this works with our first example.

#1. **Use the Echelon Method to solve the system of equations from the animal feed example at the beginning of this section.**

$$3x+10y=115$$
$$11x+4y=95$$

#2. **Use the Echelon Method to solve the system of equations.**

$$3x-4y=5$$
$$-6x+8y=7$$

#3. **Use the Echelon Method to solve the system of equations.**

$$3x-5y=2$$
$$9x-15y=6$$

In this case, we're left with a "true equation" and _____ solutions.

However, the variable *y* here is called a _____ and we define the whole solution in terms of *y*.

We can write the solution as all ordered pairs of the form

Echelon Method of Solving a Linear System

1. If possible, arrange the equations so that there is an x_1-term in the first equation, an x_2-term in the second equation, and so on.
2. Eliminate the x_1-term in all equations after the first equation.
3. Eliminate the x_2-term in all equations after the second equation.
4. Eliminate the x_3-term in all equations after the third equation.
5. Continue in this way until the last equation has the form $ax_n = k$, for constants a and k, if possible.
6. Multiply each equation by the reciprocal of the coefficient of its first term.
7. Use back-substitution to find the value of each variable.

#4. **Solve the system, and let *z* be the parameter.**

$$3x + y - z = 0$$
$$2x - y + 3z = -7$$

#5. **Kelly Karpet Kleaners sells rug cleaning machines. The EZ model weighs 10 lb and comes in a 10-cubic-ft box. The compact model weights 20 lb and comes in an 8-cubic-ft box. The commercial model weighs 60 lb and comes in a 28-cubic-ft box. Each of their delivery vans has 248 cubic ft of space and can hold a maximum of 440 lb. In order for a van to be fully loaded, how many of each model should it carry?**