10.6 – Applications: Growth & Decay
Math of Finance

Math 111

Warnock - Class Notes

Exponential Growth and Decay Function

Let y_0 be the amount or number of some quantity present at time $t = 0$. The quantity is said to grow or decay exponentially if for some constant k, the amount present at time t is given by

$$y = y_0 e^{kt}.$$

If $k > 0$, then k is called the growth constant.

If $k < 0$, then k is called the decay constant.

#1.

Giardia When a person swallows giardia cysts, stomach acids and pancreatic enzymes cause the cysts to release trophozoites, which divide every 12 hours. ***Source: The New York Times.***

a. Suppose the number of trophozoites at time $t = 0$ is y_0. Write a function in the form $y = y_0 e^{kt}$ giving the number after t hours.

b. Write the function from part a in the form $y = y_0 2^{f(t)}$.

c. The article cited above said that a single trophozoite can multiply to a million in just 10 days and a billion in 15 days. Verify this fact.

The _____ of a radioactive substance is the time it takes for exactly half of the initial quantity to decay. This is used by scientists to date remains of plants and animals

#2. **Half-Life** The half-life of radium-226 is approximately 1620 years.

a. How much of a sample weighing 4 g will remain after 100 years?

b. How much time is necessary for a sample weighing 4 g to decay to 0.1 g?

#3. **Radioactive Decay** 500 g of iodine-131 is decaying exponentially. After 3 days 386 g of iodine-131 is left.

a. Write a function in the form $y = y_0e^{kt}$ giving the number of grams of iodine-131 after t days.

b. Write the function from part a in the form $y = y_0(386/500)^{f(t)}$.

c. Use your answer from part a to find the half-life of iodine-131.

Review

Effective Rate for Compound Interest

If r is the annual stated rate of interest and m is the number of compounding periods per year, the effective rate of interest is

$$r_E = \left(1 + \frac{r}{m}\right)^m - 1.$$

Effective Rate for Continuous Compounding

If interest is compounded continuously at an annual stated rate of r, the effective rate of interest is

$$r_E = e^r - 1.$$

#4. **Effective Rate** Tami Dreyfus bought a television set with money borrowed from the bank at 9% interest compounded semiannually. What effective interest rate did she pay?

What's the effective rate if it's compounded continuously?

#5. **Interest** Greg Tobin wishes to invest a $5000 bonus check into a savings account that pays 6.3% interest. Find how many years it will take for the $5000 to grow to at least $11,000 if interest is compounded

a. quarterly. (Be careful; interest is added to the account only every quarter. See Example 5.)

b. continuously.

#6.

Population Growth The population of the world in the year 1650 was about 500 million, and in the year 2010 was 6756 million. ***Source: U.S. Census Bureau.***

a. Assuming that the population of the world grows exponentially, find the equation for the population $P(t)$ in millions in the year t.

b. Use your answer from part a to find the population of the world in the year 1.

c. Is your answer to part b reasonable? What does this tell you about how the population of the world grows?

A population of 30,000 people is growing 2.5% per year. Write an exponential function modeling this.