10.5 – Logarithmic Functions

Math 111
Warnock - Class Notes

When we did Finance Math, we talked about doubling time. For $1 to double and become $2, assuming 5% annual interest, the equation would look like this.

So far we used the graphing tool of the calculator and the TVM solver to solve this equation. However, we have ___________________________ to help solve equations like this.

Logarithm
For $a > 0$, $a \neq 1$, and $x > 0$,

$$y = \log_a x \qquad \text{means} \qquad a^y = x.$$

This reads:

These two equations interchangeable and mean the same thing!

#1. **Re-write the follow exponential equations in logarithmic form.**

$4^3 = 64$

$4^{-2} = \frac{1}{16}$

$\left(\frac{1}{3}\right)^{-4} = 81$

$e^0 = 1$

$10^3 = 1000$

#2. **Evaluate the following logarithms.**

a) $\log_3 27$

b) $\log_6 \frac{1}{36}$

c) $\log_4 (-16)$

d) $\log_2 \sqrt[3]{\frac{1}{4}}$

You Try: $\log_4 8$

Logarithmic Function

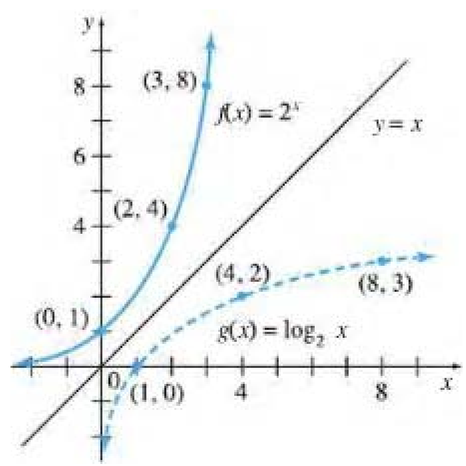
If $a > 0$ and $a \neq 1$, then the **logarithmic function** of base a is defined by

$$f(x) = \log_a x$$

for $x > 0$.

Exponential Functions and Logarithmic Functions are called ________________________ functions.

Notice:



Properties of Logarithms

Let x and y be any positive real numbers and r be any real number. Let a be a positive real number, $a \neq 1$. Then

a. $\log_a xy = \log_a x + \log_a y$

b. $\log_a \frac{x}{y} = \log_a x - \log_a y$

c. $\log_a x^r = r \log_a x$

d. $\log_a a = 1$

e. $\log_a 1 = 0$

f. $\log_a a^r = r$.

Caution: NON-Properties

Proof of a)

Let $a^m = x$ and $a^n = y$

#3. **Re-write each logarithm in the simplest possible logs using the properties of logs.**

a) $\log_7 \frac{15p}{7y}$

b) $\log_3 \frac{x+3}{y^2}$

c) $\log_2 8x^2$

From the Text, page 500:

The invention of logarithms is credited to John Napier (1550–1617), who first called logarithms "artificial numbers." Later he joined the Greek words *logos* (ratio) and *arithmos* (number) to form the word used today. The development of logarithms was motivated by a need for faster computation. Tables of logarithms and slide rule devices were developed by Napier, Henry Briggs (1561–1631), Edmund Gunter (1581–1626), and others.

For many years logarithms were used primarily to assist in involved calculations. Current technology has made this use of logarithms obsolete, but logarithmic functions play an important role in many applications of mathematics. Since our number system has base 10, logarithms to base 10 were most convenient for numerical calculations and so base 10 logarithms were called **common logarithms.** Common logarithms are still useful in other applications.

We abbreviate $\log_{10} x$ as ______________.

Most practical applications of logarithms use the number *e* as a base, so we write

$\log_e x$ **as ____________.**

Calculators have both log and ln buttons to calculate logarithms. If you want to calculate a different log base (compuer science use base 2 for example) we need the following theorem.

Change-of-Base Theorem for Logarithms

If x is any positive number and if a and b are positive real numbers, $a \neq 1$, $b \neq 1$, then

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

Proof:

$\log_b x = \log_b a^{\log_a x}$ **How?**

For any positive numbers a and x, $a \neq 1$,

$$\log_a x = \frac{\ln x}{\ln a}.$$

#4. **Evaluate** $\log_5 40$.

Solving logarithmic equations – we will use the fact that logarithms are inverses of exponential functions to solve logarithmic equations

#5. **Solve each equation.**

a) $\log_y 8 = \frac{3}{4}$

b) $\log_4 (5x+1) = 2$

c) $\log_9 m - \log_9 (m-4) = -2$

We can now use logarithms to solve Exponential Equations without having to get the bases of each side of the equation to match (as we did in the last section).

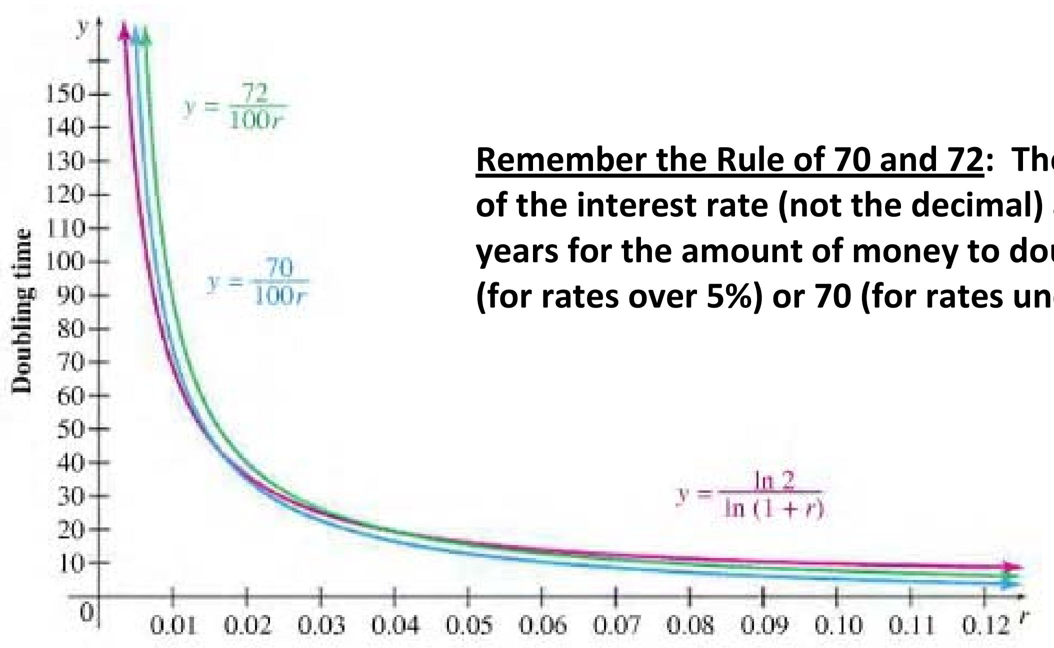
#6. **Solve the following equations.**

a) $5^x = 12$

b) $3^{x+1} = 15^x$

c) $e^{2y} = 15$

#7. **Let's go back to the problem on page one and find the amount of time it will take for money to double at 5% compounded annually.**



<u>Remember the Rule of 70 and 72</u>: The product of the interest rate (not the decimal) and the years for the amount of money to double is 72 (for rates over 5%) or 70 (for rates under 5%).

#8.

Pay Increases You are offered two jobs starting July 1, 2013. Humongous Enterprises offers you \$45,000 a year to start, with a raise of 4% every July 1. At Crabapple Inc. you start at \$30,000, with an annual increase of 6% every July 1. On July 1 of what year would the job at Crabapple Inc. pay more than the job at Humongous Enterprises? Use the algebra of logarithms to solve this problem, and support your answer by using a graphing calculator to see where the two salary functions intersect.

#9.

Drug Concentration When a pharmaceutical drug is injected into the bloodstream, its concentration at time t can be approximated by $C(t) = C_0 e^{-kt}$, where C_0 is the concentration at $t = 0$. Suppose the drug is ineffective below a concentration C_1 and harmful above a concentration C_2. Then it can be shown that the drug should be given at intervals of time T, where

$$T = \frac{1}{k} \ln \frac{C_2}{C_1}.$$

Source: Applications of Calculus to Medicine.

A certain drug is harmful at a concentration five times the concentration below which it is ineffective. At noon an injection of the drug results in a concentration of 2 mg per liter of blood. Three hours later the concentration is down to 1 mg per liter. How often should the drug be given?