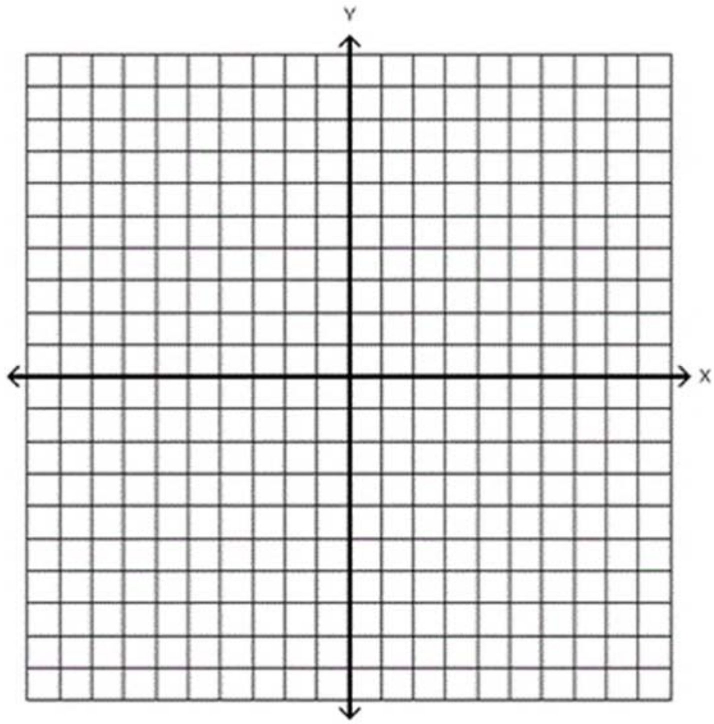
10.4 – Exponential Functions

Exponential Function

An **exponential function** with base a is defined as

$$f(x) = a^x, \quad \text{where } a > 0 \text{ and } a \neq 1.$$

Graph



$y = 2^x$

$y = 2^{-x}$

What would be another way to write this?

$y = -2^x + 3$

Exponential Equations

If $a > 0$, $a \neq 1$, and $a^x = a^y$, then $x = y$.

#1. **Solve** $8^x = 32$.

#2. **Solve** $25^x = 125^{x+2}$.

We've already worked with Exponential Functions this quarter in the form of our Compound Interest formula.

Compound Amount

If P dollars is invested at a yearly rate of interest r per year, compounded m times per year for t years, the **compound amount** is

$$A = P\left(1 + \frac{r}{m}\right)^{tm} \text{ dollars.}$$

#3. **Jennifer invests a bonus of $5000 at 5% annual interest compounded quarterly for 10 years. How much interest will she earn?**

The most important "exponential" base is the number *e*.

Let's look at our Compound interest formula for 1 dollar for 1 year at 100%.

$$P\left(1+\frac{r}{m}\right)^{tm} =$$

Now, if it only compounds annually, this will be __________

Let's look at a table for more frequent compounding.

M	$\left(1+\frac{1}{m}\right)^{m}$
4	
12	
365	
1000	

Definition of e

As m becomes larger and larger, $\left(1 + \frac{1}{m}\right)^{m}$ becomes closer and closer to the number e, whose approximate value is 2.718281828.

Now, let's revisit our Compound interest formula.

$$P\left(1+\frac{r}{m}\right)^{tm} =$$

(This last step is possible because)

And this gives us!

Continuous Compounding

If a deposit of P dollars is invested at a rate of interest r compounded continuously for t years, the compound amount is

$$A = Pe^{rt} \text{ dollars.}$$

#4. If \$1500 is invested in an account earning 3.5% compounded continuously, how much would be in the account after 8 years?

#5.

Growth of Bacteria Salmonella bacteria, found on almost all chicken and eggs, grow rapidly in a nice warm place. If just a few hundred bacteria are left on the cutting board when a chicken is cut up, and they get into the potato salad, the population begins compounding. Suppose the number present in the potato salad after x hours is given by

$$f(x) = 500 \cdot 2^{3x}.$$

a. If the potato salad is left out on the table, how many bacteria are present 1 hour later?

b. How many were present initially?

c. How often do the bacteria double?

d. How quickly will the number of bacteria increase to 32,000?

#6.

Physician Demand The demand for physicians is expected to increase in the future, as shown in the table on the following page. ***Source: Association of American Medical Colleges.***

Year	Demand for Physicians (in thousands)
2006	680.5
2015	758.6
2020	805.8
2025	859.3

a. Plot the data, letting $t = 0$ correspond to 2000. Does fitting an exponential curve to the data seem reasonable?

b. Use the data for 2006 and 2015 to find a function of the form $f(x) = Ce^{kt}$ that goes through these two points.

c. Use your function from part c to predict the demand for physicians in 2020 and 2025. How well do these predictions fit the data?

d. If you have a graphing calculator or computer program with an exponential regression feature, use it to find an exponential function that approximately fits the data. How does this answer compare with the answer to part b?