10.3 – Polynomial & Rational Functions

Math 111

Warnock - Class Notes

Polynomial Function

A **polynomial function** of **degree** n, where n is a nonnegative integer, is defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where $a_n, a_{n-1}, \ldots, a_1$, and a_0 are real numbers, called **coefficients**, with $a_n \neq 0$. The number a_n is called the **leading coefficient**.

Write the polynomial function of

Degree 1:

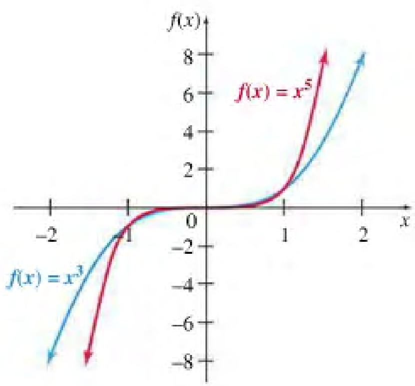
Degree 2:

Degree 3:

The simplest polynomial functions are called ________________ functions, $f(x) =$

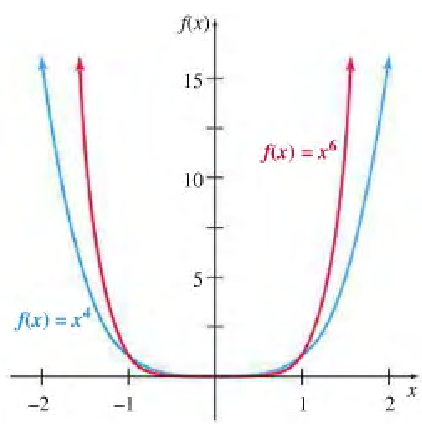
Graphs of $f(x) = x^3$ **&** $f(x) = x^5$

$f(x) = x^3$		$f(x) = x^5$	
x	$f(x)$	x	$f(x)$
−2	−8	−1.5	−7.6
−1	−1	−1	−1
0	0	0	0
1	1	1	1
2	8	1.5	7.6

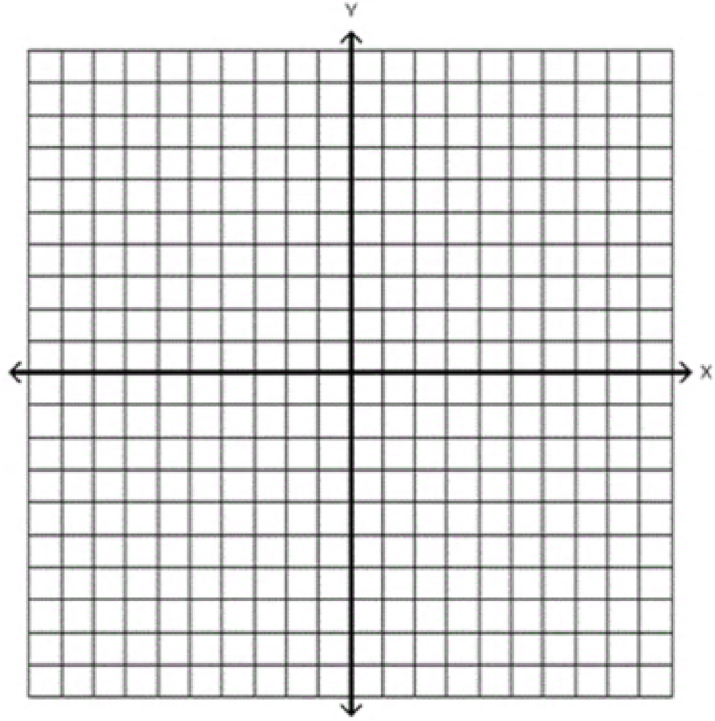


Graphs of $f(x) = x^4$ **&** $f(x) = x^6$

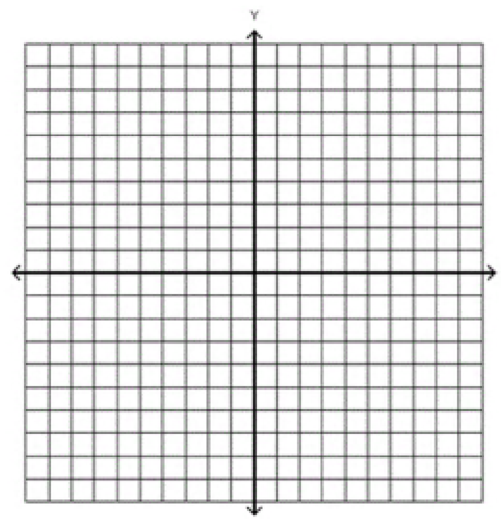
$f(x) = x^4$		$f(x) = x^6$	
x	$f(x)$	x	$f(x)$
−2	16	−1.5	11.4
−1	1	−1	1
0	0	0	0
1	1	1	1
2	16	1.5	11.4



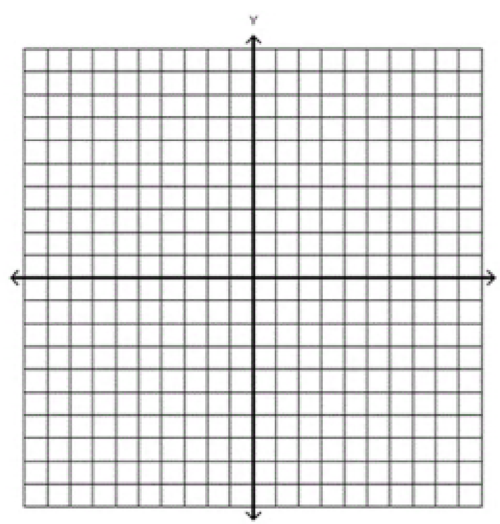
$\boxed{\textbf{\#1.}}$ **Graph** $f(x) = -(x+3)^3 - 2$



$\boxed{\textbf{\#2.}}$ **Graph** $f(x) = x^3 + x^2 - 3x - 4$ **on the graphing calculator and make observations.**



$\boxed{\textbf{\#3.}}$ **Graph** $f(x) = x^4 + 4x^3 - 3x^2 - 18x$ **on the graphing calculator and make observations.**



Domain of a polynomial function is always ________________________________.

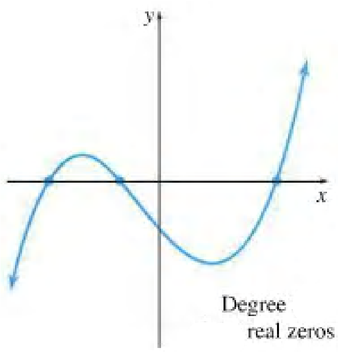
The Range of a polynomial function is

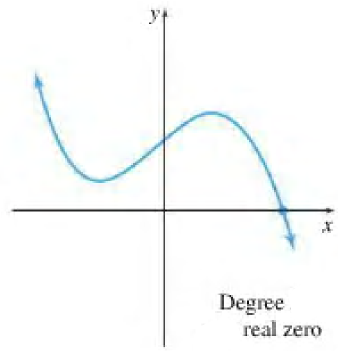
______________________ for odd degree.

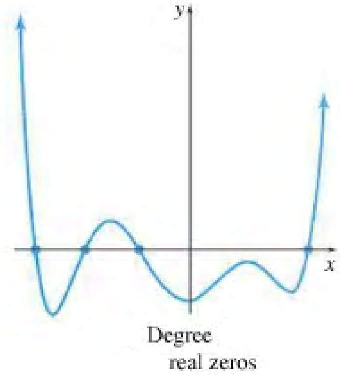
_________________ ___ ____ /____ _ for even degree.

The number of "roots" or "zeros" is ___________________________ the degree of the polynomial.

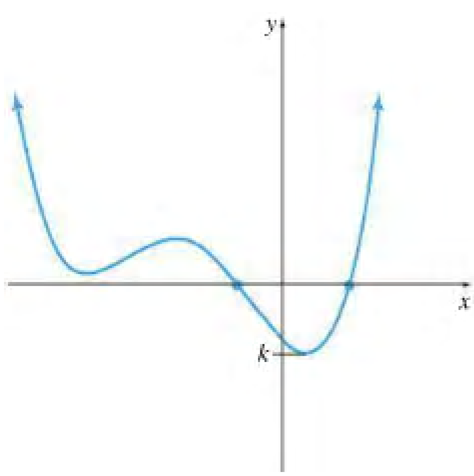
The number of "turns" is _________________________________ the degree of the polynomial.

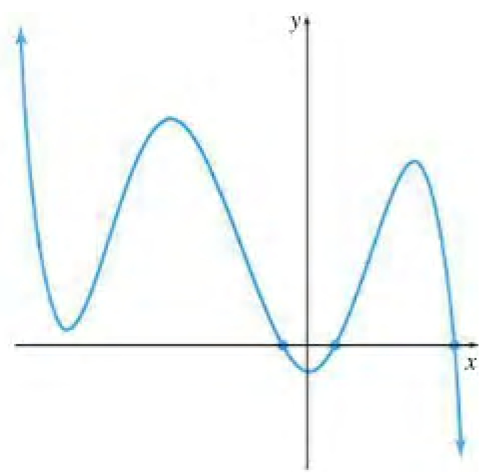






#4. **Identify the degree of each polynomial pictured, and give the sign (+ or -) for the leading coefficient.**





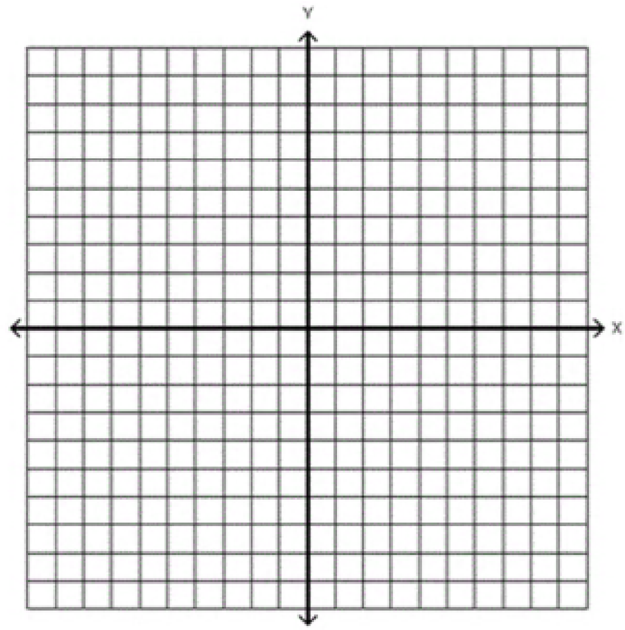
Rational Function

A **rational function** is defined by

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$.

$\boxed{\#5.}$ **Graph** $f(x) = \frac{1}{x}$



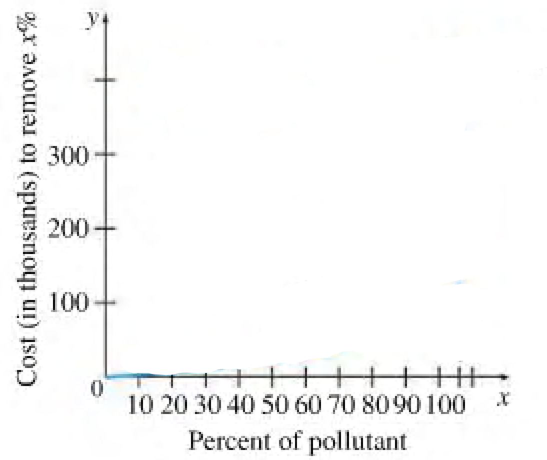
Asymptotes

If a function gets larger and larger in magnitude without bound as x approaches the number k, then the line $x = k$ is a **vertical asymptote**.

If the values of y approach a number k as $|x|$ gets larger and larger, the line $y = k$ is a **horizontal asymptote**.

Rational functions occur often in practical applications. In many situations involving environmental pollution, much of the pollutant can be removed from the air or water at a fairly reasonable cost, but the last small part of the pollutant can be very expensive to remove. Cost as a function of the percentage of pollutant removed from the environment can be calculated for various percentages of removal, with a curve fitted through the resulting data points. This curve then leads to a mathematical model of the situation. Rational functions are often a good choice for these **cost-benefit models** because they rise rapidly as they approach a vertical asymptote.

$\boxed{\#6.}$ **Suppose a cost-benefit model is given by $f(x)=\dfrac{18x}{106-x}$ where y is the cost (in thousands of dollars) of removing x percent of a certain pollutant. The domain os x is the set of all numbers from 0 to 100 inclusive; any amount of pollutant from 0% to 100% can be removed. Find the cost to remove the following amounts of the pollutant: 100%, 95%, 90%, and 80%. Graph the function.**



If a cost function has the form $C(x) = mx + b$, where x is the number of items produced, m is the marginal cost per item and b is the fixed cost, then the **average cost** per item is given by

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{mx + b}{x}.$$

Notice that this is a rational function with a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = m$. The vertical asymptote reflects the fact that, as the number of items produced approaches 0, the average cost per item becomes infinitely large, because the fixed costs are spread over fewer and fewer items. The horizontal asymptote shows that, as the number of items becomes large, the fixed costs are spread over more and more items, so most of the average cost per item is the marginal cost to produce each item. This is another example of how asymptotes give important information in real applications.

$\boxed{\#7.}$ **Suppose the average cost per unit $\overline{C}(x)$, in dollars, to produce x units of yogurt is given by $\overline{C}(x)=\dfrac{600}{x+20}$.**

a) **Find and interpret $\overline{C}(100), \overline{C}(50), \overline{C}(10), \& \ \overline{C}(1)$.**
b) **Explain why $\overline{C}(0)$ doesn't make sense to discuss.**