$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

Method: Addition and subtraction with like denominators

To add or subtract when denominators are the same, add or subtract the numerators and keep the same denominator.

$$\frac{A}{C} + \frac{B}{C} = \frac{A+B}{C} \text{ and } \frac{A}{C} - \frac{B}{C} = \frac{A-B}{C}, \text{ where } C \neq 0$$

Example 1: Add or subtract

a.) $\frac{5}{3a} + \frac{7}{3a} = \boxed{\frac{12}{3a}}$

b.) $\frac{a-5b}{a+b} + \frac{a+7b}{a+b} = \frac{a-5b+a+7b}{a+b} = \frac{2a+2b}{a+b}$

$= \frac{2\cancel{(a+b)}}{\cancel{(a+b)}}$

$= \boxed{2}$

c.) $\frac{4y+2}{y-2} - \frac{y-3}{y-2} = \frac{4y+2-(y-3)}{y-2} = \frac{4y+2-y+3}{y-2}$

$= \boxed{\frac{3y+5}{y-2}}$

Method: The Least Common Multiple

To find the least common multiple (LCM) of two or more expressions, find the prime factorization of each expression and form a product that contains each factor the greatest number of times that it occurs in any one prime factorization.

LCM: 12 & 16

16, 32, 48

12: 4, 3; 2, 2

$2^2 \cdot 3$

16: 2^4

LCM: $2^4 \cdot 3$

Example 2: Find the LCM

a.) $24x^2y$ and $9xy^4$

$2^3 \cdot 3$ $\qquad$ 3^2

LCM: $2^3 \cdot 3^2 \cdot x^2 \cdot y^4$

$\boxed{72x^2y^4}$

b.) $t^2 - 25$ and $t^2 - 10t + 25$

$(t-5)(t+5)$ and $(t-5)^2$

LCM: $(t-5)^2(t+5)$

<u>Method</u>: To add or subtract rational expressions

1.) Determine the least common denominator (LCD) by finding the least common multiple of the denominators.
2.) Rewrite each of the original rational expressions, as needed, in an equivalent form that has the LCD. Multiply be "special 1's" to get denominators to the LCD
3.) Add or subtract the resulting rational expressions, as indicated.
4.) Simplify the result, if possible, and list any restrictions on the domain of the functions.

Example 3: Add or subtract. Always simplify if possible.

$\frac{2}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{3} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

a.) $\frac{a+3}{a-5} + \frac{a-2}{a+4}$

LCD: ~~$-5\ 4\ a$~~

$(a-5)(a+4)$

$\frac{(a+4)}{(a+4)} \cdot \frac{(a+3)}{(a-5)} + \frac{(a-2)}{(a+4)} \cdot \frac{(a-5)}{(a-5)}$

$\frac{a^2+7a+12}{(a+4)(a-5)} + \frac{a^2-7a+10}{(a+4)(a-5)} = \frac{a^2 + \cancel{7a} + 12 + a^2 - \cancel{7a} + 10}{(a+4)(a-5)}$

$= \boxed{\frac{2a^2+22}{(a+4)(a-5)} = \frac{2(a^2+11)}{(a+4)(a-5)}}$

b.) $\frac{a+3}{5a+25} - \frac{a-1}{3a+15}$

LCD: $3 \cdot 5 \cdot (a+5)$

$15(a+5)$

$$\frac{3}{3} \cdot \frac{a+3}{5(a+5)} - \frac{a-1}{3(a+5)} \cdot \frac{5}{5}$$

$$\frac{3(a+3) - 5(a-1)}{15(a+5)} = \frac{3a+9-5a+5}{15(a+5)}$$

$$= \boxed{\frac{-2a+14}{15(a+5)} = \frac{2(-a+7)}{15(a+5)}}$$

c.) $\frac{7}{3y^2+y-4} + \frac{9y+2}{3y^2-2y-8}$

$(3y+4)(y-1)$ $\quad$ $(3y+4)(y-2)$

LCD: $(3y+4)(y-1)(y-2)$

$$\frac{(y-2)}{(y-2)} \cdot \frac{7}{(3y+4)(y-1)} + \frac{(9y+2)}{(3y+4)(y-2)} \cdot \frac{(y-1)}{(y-1)}$$

$$= \frac{\cancel{7y} - 14 + 9y^2 \cancel{-9y} \cancel{+2y} - 2}{(3y+4)(y-1)(y-2)} = \frac{9y^2-16}{(3y+4)(y-1)(y-2)}$$

$$= \frac{(3y-4)\cancel{(3y+4)}}{\cancel{(3y+4)}(y-1)(y-2)}$$

$$= \boxed{\frac{3y-4}{(y-1)(y-2)}}$$

d.) $\frac{m-3n}{m^3-n^3}-\frac{2n}{n^3-m^3} = \frac{m-3n}{(m^3-n^3)} - \frac{2n}{-(m^3-n^3)} = \frac{m-3n+2n}{(m^3-n^3)}$

$\cancel{\frac{m-3n}{(m-n)(m^2+mn+n^2)} - \frac{2n}{(n-m)(n^2+nm+m^2)}}$

$-(m-n)$

$\frac{\cancel{(m-n)}}{\cancel{(m-n)}(m^2+mn+n^2)}$

$\boxed{\frac{1}{m^2+mn+n^2}}$

e.) $\frac{-2}{y+2}+\frac{5}{y-2}+\frac{y+3}{y^2-4}$

$(y-2)(y+2)$

LCD: $(y+2)(y-2)$

~~equations~~

$\frac{(y-2)}{(y-2)}\cdot\frac{-2}{(y+2)} + \frac{5}{(y-2)}\cdot\frac{(y+2)}{(y+2)} + \frac{y+3}{(y-2)(y+2)}$

$\frac{-2y+4+5y+10+y+3}{(y-2)(y+2)} = \boxed{\frac{4y+17}{(y-2)(y+2)}}$

f.) $\dfrac{5x}{x^2-6x+8}-\dfrac{3x}{x^2-x-12}$

$(x-4)(x-2)$ $(x-4)(x+3)$

LCD: $(x-4)(x-2)(x+3)$

$$\frac{5x}{(x-4)(x-2)}\cdot\frac{(x+3)}{(x+3)}-\frac{3x}{(x-4)(x+3)}\cdot\frac{(x-2)}{(x-2)}$$

$$\frac{5x^2+15x-3x^2+6x}{(x-4)(x-2)(x+3)}=\boxed{\frac{2x^2+21x}{(x-4)(x-2)(x+3)}}$$

g.) $\dfrac{x-1}{x^2-1}-\dfrac{x}{x-2}+\dfrac{x^2+2}{x^2-x-2}$

$(x-1)(x+1)$ $(x-2)(x+1)$

$$\frac{(x-1)(x+1)}{(x-1)(x+1)}=\frac{x^2-1}{x^2-1}$$

LCD: $(x-1)(x+1)(x-2)$

$x(x+1)(x-1)$

$\cancel{x^2+x+x^2-x}$

$(x^2+x)(x-1)$

$x^3\cancel{-x^2}\cancel{+x^2}-x$

$$\frac{(x-2)}{(x-2)}\frac{(x-1)}{(x-1)(x+1)}-\frac{x}{(x-2)}\cdot\frac{(x^2-1)}{(x^2-1)}+\frac{(x^2+2)}{(x-2)(x+1)}\cdot\frac{(x-1)}{(x-1)}$$

$$\frac{\cancel{x^2}\cancel{-2x}\cancel{-x}\cancel{+2}\cancel{-x^3}\cancel{+x}\cancel{+x^3}\cancel{-x^2}\cancel{+2x}\cancel{-2}}{(x-2)(x-1)(x+1)}=\frac{0}{\quad}$$

$=\boxed{0}$

LCD $(y-3)(y+3)(y-1)$

$$\frac{(y-1)}{(y-1)}\cdot\frac{(2y-6)}{\underbrace{(y^2-9)}_{(y-3)(y+3)}}+\frac{(y-3)}{(y-3)}\frac{(y^2+2)}{\underbrace{y^2+2y-3}_{(y+3)(y-1)}}-\frac{y}{y-1}\cdot\frac{(y+3)(y-3)}{(y+3)(y-3)} \text{ or } \frac{y^2-9}{y^2-9}$$

$$\frac{2y^2-6y-2y+6+y^3+2y-3y^2-6-y^3+9y}{(y-3)(y+3)(y-1)}$$

$$\frac{2y^2-6y\cancel{-2y}\cancel{+6}\cancel{+y^3}\cancel{+2y}-3y^2\cancel{-6}\cancel{-y^3}+9y}{(y-3)(y+3)(y-1)}=\frac{-y^2+3y}{}$$

$$=\frac{-y\,(\cancel{y-3})}{(\cancel{y-3})(y+3)(y-1)}$$

$$=\boxed{\frac{-y}{(y+3)(y-1)}}$$