8.9 - Polynomial & Rational Inequalities

Note Title 3/4/2009

Quadratic Inequalities

Graph $y = x^2 - 3x - 10$

When is $y = 0$?

$y < 0$?

$y > 0$?

$x^2 - 3x - 10 = (x-5)(x+2) = 0$ divides the graph into 3 regions

Test Points	$(x-5)(x+2)$

(1) Solve $x^2 + 2x - 8 > 0$

② Solve $(x+3)(x-4)(x-1) \leq 0$

③ Solve $x^4 - x^3 - 12x^2 < 0$

To Solve a Polynomial Inequality

1. Add or subtract to get 0 on one side and solve the related polynomial equation $p(x) = 0$.
2. Use the numbers found in step (1) to divide the number line into intervals.
3. Using a test value from each interval or the graph of the related function, determine the sign of $p(x)$ over each interval.
4. Select the interval(s) for which the inequality is satisfied and write set-builder notation or interval notation for the solution set. Include the endpoints of the intervals when $\leq$ or $\geq$ is used.

(4) Solve $\frac{x-3}{x+5} \leq 0$

(5) Solve $\frac{x-1}{x-4} \geq -2$

To Solve a Rational Inequality

1. Change the inequality symbol to an equals sign and solve the related equation.
2. Find any replacements for which the rational expression is undefined.
3. Use the numbers found in steps (1) and (2) to divide the number line into intervals.
4. Substitute a test value from each interval into the inequality. If the number is a solution, then the interval to which it belongs is part of the solution set.
5. Select the interval(s) and any endpoints for which the inequality is satisfied and write set-builder or interval notation for the solution set. If the inequality symbol is $\leq$ or $\geq$, then the solutions from step (1) are also included in the solution set. Those numbers found in step (2) should be excluded from the solution set, even if they are solutions from step (1).

(6) Solve $\dfrac{(x+2)(x+4)}{(x-1)^2} \geq 0$