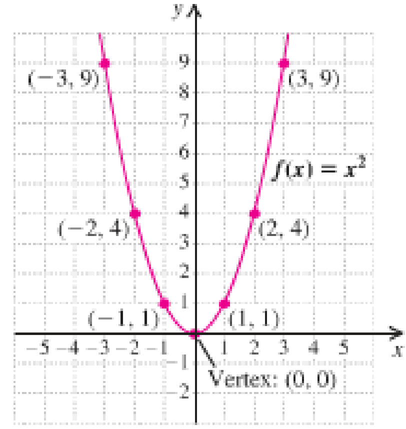
8.6-8.7 — Quadratic Functions & Their Graphs

Note Title

$f(x)=x^2$

x	$f(x)=x^2$	$(x, f(x))$
-3	9	$(-3, 9)$
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	4	$(2, 4)$
3	9	$(3, 9)$



$F(x)=a(x-h)^2+K$

shift ______ or ______, ______ as the sign you see.

shift ______ or ______, ______ of the sign you see.

______ $(a<1)$ or ______ $(a>1)$

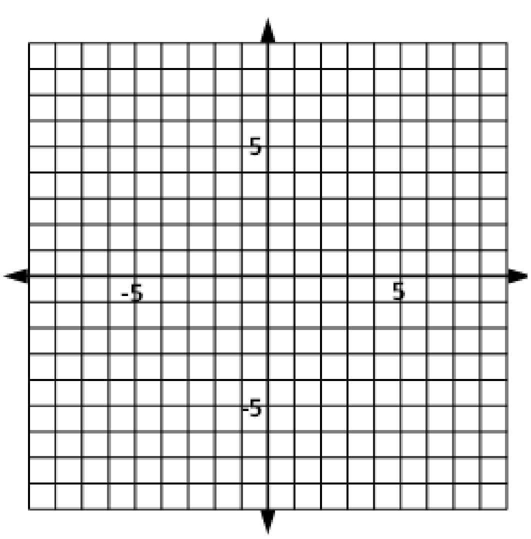
______ $(a>0)$ or ______ $(a<0)$.

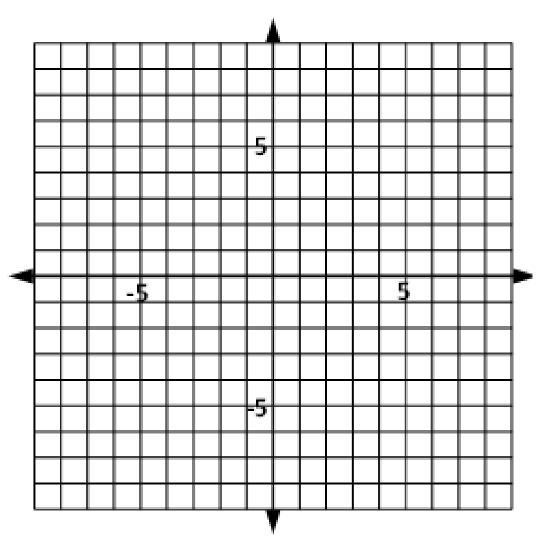
* Read 8.6 if you'd like a more rigorous development of these ideas.

① Graph accurately.

a) $f(x)=\frac{1}{2}(x+4)^2-5$

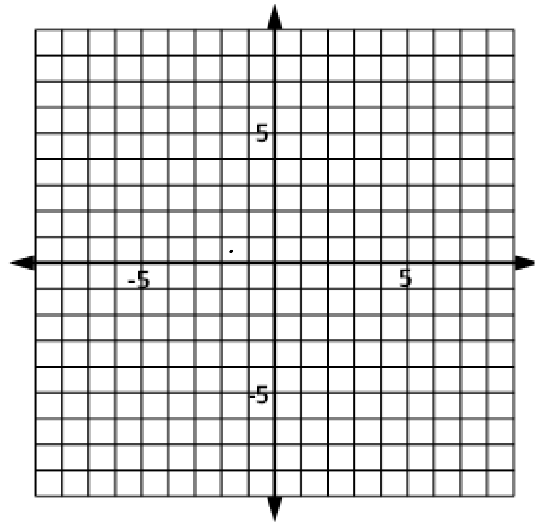
b) $g(x)=-3(x-2)^2+3$



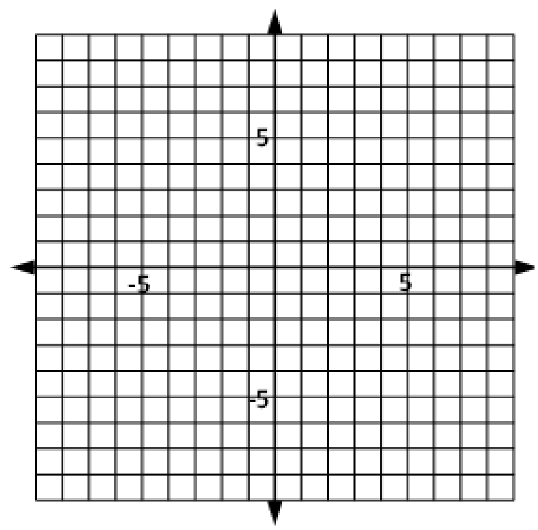


(8.7)

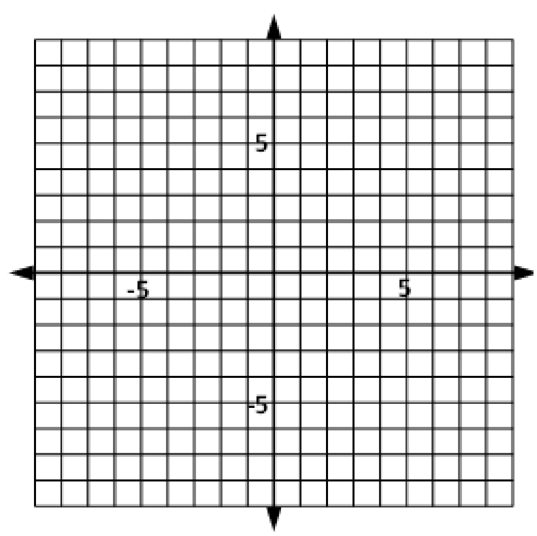
② Graph $f(x) = x^2 - 8x + 9$ by completing the square.



③ Graph $f(x) = 2x^2 + 4x + 6$ by completing the square.



④ Graph $f(x) = -\frac{1}{2}x^2 + 3x - \frac{1}{2}$ by completing the square.



Find the vertex by completing the square.

$F(x) = ax^2 + bx + c$

The Vertex of a Parabola The vertex of the parabola given by $f(x) = ax^2 + bx + c$ is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right), \quad \text{or} \quad \left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right).$$

The x-coordinate of the vertex is $-b/(2a)$. The axis of symmetry is $x = -b/(2a)$. The second coordinate of the vertex is most commonly found by computing $f\left(-\frac{b}{2a}\right)$.

5 Find the vertex

$g(x) = 2x^2 + 5x - 1$

(Use Graphing Calc to check)

⑥ For the following functions, find the vertex, all intercepts, and max or min value.

a) $f(x) = 4x^2 - 12x + 3$

b) $g(x) = -18.8x^2 + 7.92x + 6.18$

The Graph of a Quadratic Function Given by $f(x) = ax^2 + bx + c$ or $f(x) = a(x - h)^2 + k$

The graph is a parabola.

The vertex is (h, k) or $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The axis of symmetry is $x = h$.

The y-intercept of the graph is $(0, c)$.

The x-intercepts can be found by solving $ax^2 + bx + c = 0$.

- If $b^2 - 4ac > 0$, there are two x-intercepts.
- If $b^2 - 4ac = 0$, there is one x-intercept.
- If $b^2 - 4ac < 0$, there are no x-intercepts.

The domain of the function is $(-\infty, \infty)$.

If a is positive: The graph opens upward.
The function has a minimum value, given by k.
This occurs when $x = h$.
The range of the function is $[k, \infty)$.

If a is negative: The graph opens downward.
The function has a maximum value, given by k.
This occurs when $x = h$.
The range of the function is $(-\infty, k]$.