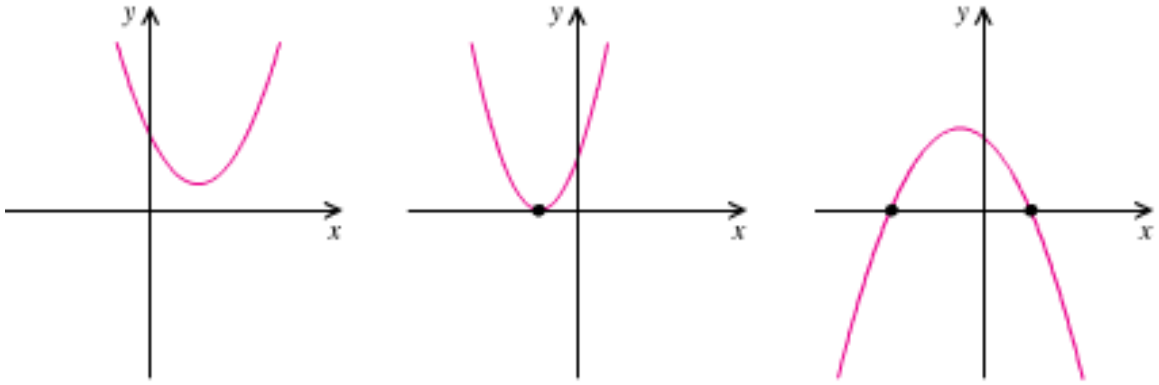
8.1 - Quadratic Equations

Note Title

As seen in Math 91 and earlier this quarter, the graphs of Quadratic equations are parabolas

When solving a quadratic, there are 3 cases.



No x-intercepts
No real-number roots

One x-intercept
One real-number root

Two x-intercepts
Two real-number roots

① Solve $6x^2 = x + 12$

② Solve $x^2 = 49$

Intuitively, how could we solve $x^2 = 49$?

The Principle of Square Roots For any real number k, if $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$.

Another notation ⇒

(3) Solve

a) $4x^2 = 20$

b) $-7x^2 + 6 = 0$

c) $9x^2 + 10 = 0$

The Principle of Square Roots (Generalized Form) For any real number k and any algebraic expression X,

If $X^2 = k$, then $X = \sqrt{k}$ or $X = -\sqrt{k}$.

④ Let $f(x) = (x+3)^2$, find all x's where $F(x) = 6$.

(Algebraically & Graphically)

⑤ Solve $x^2 - 10x + 25 = 3$

Review

$x^2 + 8x + 16 = (x \quad)^2$

$x^2 - 10x + 25 = (x \quad)^2$

$x^2 - 7x + \frac{49}{4} = (x \quad)^2$

Completing the Square

⑥ Solve $x^2+6x-2=0$

⑦ What number should be used to "Complete the square".

a) $x^2+12x+\_\_\_\_\_=(x+\_\_\_\_\_)^2$

b) $x^2-3x+\_\_\_\_\_=(x-\_\_\_\_\_)^2$

c) $x^2-\frac{4}{3}x+\_\_\_\_\_=(x-\_\_\_\_\_)^2$

⑧ Solve $x^2-10x-3=0$ by Completing the Square.

To Solve a Quadratic Equation in x by Completing the Square

1. Isolate the terms with variables on one side of the equation, and arrange them in descending order.
2. Divide both sides by the coefficient of x^2 if that coefficient is not 1.
3. Complete the square by taking half of the coefficient of x and adding its square to both sides.
4. Express the trinomial as the square of a binomial (factor the trinomial) and simplify the other side.
5. Use the principle of square roots (find the square roots of both sides).
6. Solve for x by adding or subtracting on both sides.

(9) Solve $4x^2 + 3x - 20 = 0$

(10) Find the x-intercepts of $y = 2x^2 - 5x - 3$

The Compound-Interest Formula If an amount of money P is invested at interest rate r, compounded annually, then in t years, it will grow to the amount A given by

$$A = P(1 + r)^t. \qquad (r \text{ is written in decimal notation.})$$

(11) Find the interest rate if \$6250 invested grows to \$7290 in 2 years.

EXAMPLE 13 Free-Falling Objects. The formula $s = 16t^2$ is used to approximate the distance s, in feet, that an object falls freely from rest in t seconds. Ireland's Cliffs of Moher are 702 ft tall (*Source*: Based on data from 4windstravel.com). How long will it take a stone to fall from the top? Round to the nearest tenth of a second.