7.2 - Rational Exponents

Note Title

$a^{1/n} = \sqrt[n]{a}$ $a^{1/n}$ means $\sqrt[n]{a}$. When a is nonnegative, n can be any natural number greater than 1. When a is negative, n must be odd.

(1) Write in radical notation & simplify.

a) $x^{1/2}$ b) $(-27)^{1/3}$ c) $(365^{12})^{1/2}$

(2) Write with exponential notation.

a) $\sqrt[4]{7ab}$ b) $\sqrt[5]{\frac{3x}{7y}}$

(3) Graph $f(x) = \sqrt[4]{3x-2}$ on calculator.

Positive Rational Exponents For any natural numbers m and n ($n \neq 1$) and any real number a for which $\sqrt[n]{a}$ exists,

$$a^{m/n} \text{ means } (\sqrt[n]{a})^m, \text{ or } \sqrt[n]{a^m}.$$

(4) Write in radical notation & simplify

a) $8^{2/3}$

b) $36^{3/2}$

Negative Rational Exponents For any rational number m/n and any nonzero real number a for which $a^{m/n}$ exists,

$$a^{-m/n} \text{ means } \frac{1}{a^{m/n}}.$$

(5) Write with positive exponents & simplify if possible

a) $49^{-1/2}$

b) $(-27)^{-2/3}$

c) $5a^{-3/2}b^{4/3}$

d) $\left(\frac{x}{y}\right)^{-3/5}$

Laws of Exponents For any real numbers a and b and any rational exponents m and n for which a^m, a^n, and b^m are defined:

1. $a^m \cdot a^n = a^{m+n}$ — In multiplying, add exponents if the bases are the same.
2. $\frac{a^m}{a^n} = a^{m-n}$ — In dividing, subtract exponents if the bases are the same. (Assume $a \neq 0$.)
3. $(a^m)^n = a^{m \cdot n}$ — To raise a power to a power, multiply the exponents.
4. $(ab)^m = a^m b^m$ — To raise a product to a power, raise each factor to the power and multiply.

(6) Simplify

a) $5^{3/7} \cdot 5^{1/7}$

b) $\frac{a^{1/6}}{a^{1/2}}$

c) $\left(\pi^{3/4}\right)^{2/3}$

d) $\left(r^{-1/4} b^{3/7}\right)^{1/3}$

To Simplify Radical Expressions

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

(7) Simplify

a) $\sqrt[4]{5^{12}}$

b) $\left(\sqrt[5]{x^2 y}\right)^{20}$

c) $\sqrt[8]{(3y)^4}$

d) $\sqrt[3]{\sqrt{r}}$