6.1 – Rational Expressions & Functions $(\cdot, \div)$

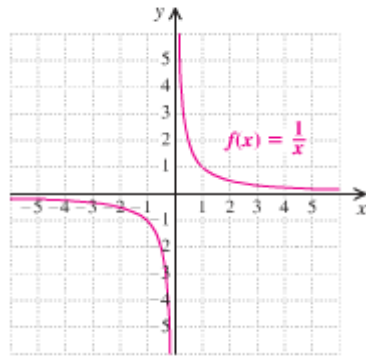Rational Expressions – an expression
consisting of a _____ _____ by a
_____ _____.

Ex:

Rational expressions can be used to describe
certain _____, called _____ _____.

Ex: $f(x) = \dfrac{1}{x}$

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① Photo Developing.   Rik usually takes 3 hr more than Pearl does to process a day's orders at Liberty Place Photo. If Pearl takes t hr to process a day's orders, the function given by

$$H(t) = \frac{t^2 + 3t}{2t + 3}$$

can be used to determine how long it would take if they worked together.

How long will it take them, working together, to complete a day's orders if Pearl can process the orders alone in 5 hr?

Products of Rational Expressions To multiply two rational expressions, multiply numerators and multiply denominators:

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}, \quad \text{where } B \neq 0, D \neq 0.$$

② Multiply

$$\frac{x+5}{x-4} \cdot \frac{y-1}{x^3}$$

Review: How do we reduce $\frac{8}{14}$?

Similarly, reduce $\frac{(x-3)(x-2)}{(x+5)(x-2)} =$

What is different between

$$\frac{(x-3)(x-2)}{(x+5)(x-2)} \neq \frac{x-3}{x+5}$$

③ Write the function in simplified form. (Be careful with domain.)

$$\left(\begin{array}{c}\downarrow \text{no more} \\ \text{factors of} \\ \text{"1" can be} \\ \text{removed}\end{array}\right)$$

a) $f(t) = \dfrac{5x^2 + 20x}{x^2 + 4x}$

b) $g(m) = \dfrac{m^2 - 9}{3m + 3} \cdot \dfrac{m + 3}{m - 3}$

④ Simplify

a) $\dfrac{x^2 - 16}{x^2} \cdot \dfrac{x^2 - 4x}{x^2 - x - 12}$

b) $\dfrac{a^2-1}{2-5a} \cdot \dfrac{15a-6}{a^2+5a-6}$

We <u>can not</u> cancel over _____

Ex: $\dfrac{x+1}{x}$ $\qquad)\qquad$ $\dfrac{6t-1}{2}$

c) $\dfrac{x^3+y^3}{x^2+2xy-3y^2} \cdot \dfrac{x^2-y^2}{3x^2+6xy+3y^2}$

Quotients of Rational Expressions For any rational expressions A/B and C/D, with $B, C, D \neq 0$,

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C}.$$

(To divide two rational expressions, multiply by the reciprocal of the divisor. We often say that we "*invert* and multiply.")

⑤ Divide # Simplify

a) $\dfrac{3y+15}{y^7} \div \dfrac{y+5}{y^2}$

b) $\dfrac{x^2-y^2}{4x+4y} \div \dfrac{3y-3x}{12x^2}$

⑥ Simplify the function, list all domain restrictions.

$$g(x) = \frac{x^2-9}{x^2} \div \frac{x^5+3x^4}{x+2}$$

Vertical Asymptotes

Consider

$$H(t) = \frac{t^2 + 5t}{2t + 5}$$

For Domain, we know...?

Looking at the graph, what happens at $t = -\frac{5}{2}$?

This is called a ___ ___

Examine

$$f(x) = \frac{(x-1)(x+3)}{(2x+1)(x+3)} \quad \text{and} \quad g(x) = \frac{x-1}{2x+1}$$

Find domains & vertical asymptotes

⑦ Find the vertical asymptotes.

$$g(x) = \frac{x^2 - 4}{2x^2 - 5x + 2}$$