5.1 & 5.5 - Exponents

Note Title

ZERO EXPONENT

For any nonzero real number b,

$$b^0 = 1.$$

The expression 0^0 is undefined.

Let's observe the pattern.

$3^4 =$

$3^3 =$

$3^2 =$

$3^1 =$

$3^0 =$

What does this mean?

$x^3 \cdot x^4$

THE PRODUCT RULE

For any real number a and natural numbers m and n,

$$a^m \cdot a^n = a^{m+n}.$$

What does this mean?

$(x^3)^4$

RAISING A POWER TO A POWER

For any real number a and natural numbers m and n,

$$(a^m)^n = a^{mn}.$$

① Simplify

a) -6^2

b) $(-6)^2$

c) 3^0

d) $5\left(\frac{2}{3}\right)^0$

e) $3^2 \cdot 3^4$

f) $x^5 \cdot x^8$

g) $3y^7 \cdot 4y^4$

h) $2x^3(3x^4 - 5x^7)$

i) $(2^2)^3$

j) $(x^5)^7$

What does this mean?

$(xy)^3$

$\left(\frac{x}{y}\right)^3$

RAISING A PRODUCT TO A POWER

For any real numbers a and b and natural number n,

$$(ab)^n = a^n b^n.$$

RAISING A QUOTIENT TO A POWER

For any real numbers a and b and natural number n,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}. \qquad b \neq 0$$

(2) Simplify

a) $(4y)^2$

b) $(-3x^4)^3$

c) $5(a^3b^4)^2$

d) $\left(\frac{2}{3}\right)^3$

e) $\left(\frac{r^4s^5}{t^2}\right)^3$

f) $(2x^2y)^3(-3x^3y^5)^2$

Let's observe the pattern again, but Further.

$3^4 =$

$3^3 =$

$3^2 =$

$3^1 =$

$3^0 =$

$3^{-1} =$

$3^{-2} =$

$3^{-3} =$

NEGATIVE INTEGER EXPONENTS

Let a be a nonzero real number and n be a positive integer. Then

$$a^{-n} = \frac{1}{a^n}.$$

That is, a^{-n} is the reciprocal of a^n.

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(3) Simplify

a) 3^{-2}

b) 5^{-1}

c) x^{-3}

d) $4^3 \cdot 4^{-5}$

e) $5^{-2} \cdot 5^{-1}$

f) $x^{-3} \cdot x^6$

g) $(r^2)^{-3}$

h) $(ab^{-2})^2 (ab)^{-3}$

What does this mean?

$$\frac{x^7}{x^3}$$

THE QUOTIENT RULE

For any nonzero number a and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n}.$$

(4) Simplify

a) $\frac{5^2}{5^5}$

b) $\frac{12c^9}{4c^2}$

c) $\frac{a^2b^5}{a^4b}$

QUOTIENTS AND NEGATIVE EXPONENTS

The following three rules hold for any nonzero real numbers a and b and positive integers m and n.

1. $\frac{1}{a^{-n}} = a^n$

2. $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

3. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

(5) Simplify

a) $\frac{3^{-2}}{5^{-3}}$

b) $\frac{4x^{-3}y^5}{14x^4y^{-2}}$