12.2 - Exponential Functions

Note Title

Many things in life don't grow or decrease at a constant rate (linear). The amount of money in an account will affect the amount of interest earned each following year.

For example, at 5% compounded annually

$100 will earn ________ in 1 year

________ in 2 years

________ in 5 years

This can be modeled with Exponential Functions.

EXPONENTIAL FUNCTION

A function represented by

$$f(x) = Ca^x, \quad a > 0 \quad \text{and} \quad a \neq 1,$$

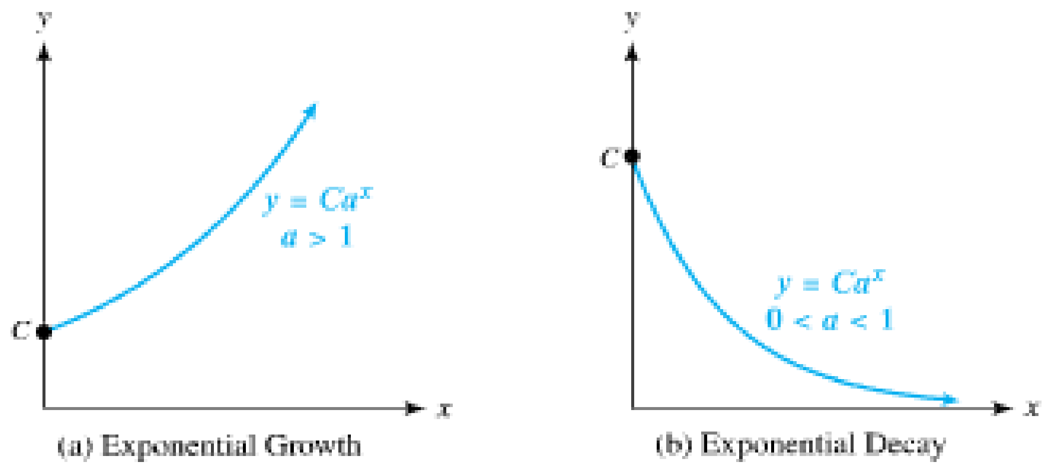
is an **exponential function with base *a* and coefficient *C***. (Unless stated otherwise, we assume that $C > 0$.)

C is the y-intercept since

$f(0) =$

If $a>1$, then we have ______ ____.

If $0<a<1$, then we have ______ ____.



1. Evaluate the functions at the given input

a) $f(x) = 5(2)^x$, $x=3$

b) $g(t) = 4\left(\frac{1}{2}\right)^x$, $x=4$

c) $P(r) = \frac{1}{2}(3)^x$, $x=-2$

a^{-x} and $\left(\frac{1}{a}\right)^x$

② Fill in the following table, evaluating the functions at the given values. (You may use a calculator!)

x	$f(x)=3^x$	$g(x)=\left(\frac{2}{5}\right)^x$	$h(x)=6(2)^x$	$j(x)=10(1.07)^x$
10				
3				
1				
0				
–2				

③ Determine whether the following functions are "exponential," "linear", or neither. Can you write the function?

x	$n(x)$	$p(x)$	$q(x)$	$r(x)$	$s(x)$
0	243	39	16	5	12
1	81	31	24	11	16.75
2	27	23	36	18	21.5
3	9	15	54	26	26.25
4	3	7	81	35	31
5	1	–1	121.5	45	35.75
Type:					

Function?:

4

To understand exponential growth better, it's helpful to compare it with linear growth. Suppose you are offered jobs at two different companies, and when discussing your pay, Company A offers you a starting salary of $50,000 with annual raises of $2000 (assuming you perform well). Company B offers the same starting salary, but says your raises will be 4% per year. Complete the following table to see how each salary grows.

Company A			Company B	
Year Employed	Annual Salary		Year Employed	Annual Salary
1	$50,000		1	$50,000
2			2	
3			3	
4			4	
5			5	
6			6	
		Pattern/Formula		

COMPOUND INTEREST

If C dollars are deposited in an account and if interest is paid at the end of each year with an annual rate of interest r, expressed in decimal form, then after x years the account will contain A dollars, where

$$A = C(1 + r)^x.$$

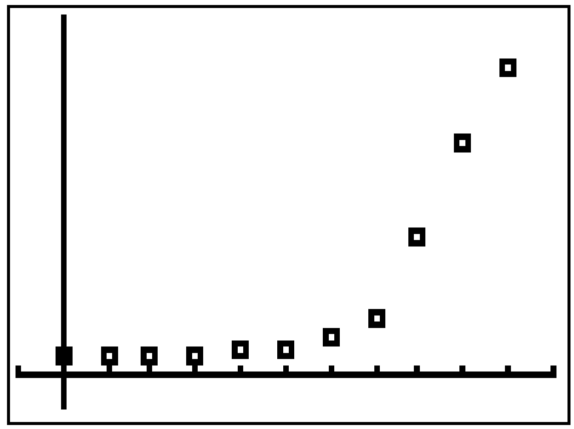
The growth factor is $(1 + r)$.

5 Compound Interest problem

⑥ Look at the following real world data that can be modeled with exponential functions and plot the points.

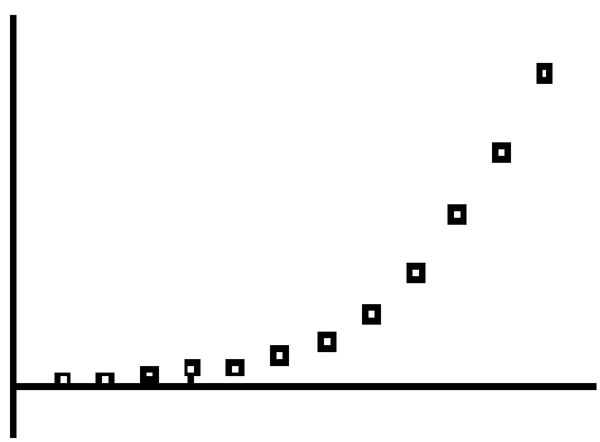
Price of 1st Class Stamp

Year	Stamp Price
1900	0.02
1910	0.02
1920	0.02
1930	0.02
1940	0.03
1950	0.03
1960	0.04
1970	0.06
1980	0.15
1990	0.25
2000	0.33

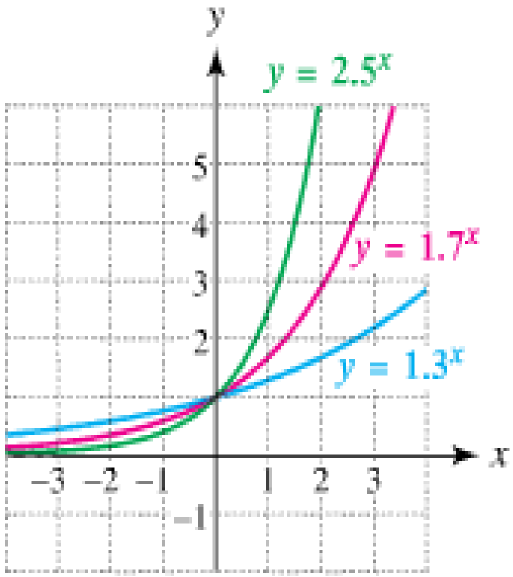


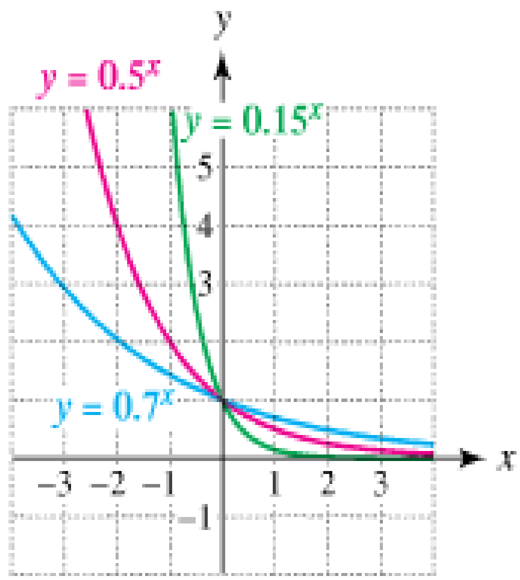
Number of Starbucks Stores worldwide.

Year	Number of Stores
1987	17
1988	33
1989	55
1990	84
1991	116
1992	165
1993	272
1994	425
1995	676
1996	1015
1997	1412
1998	1886



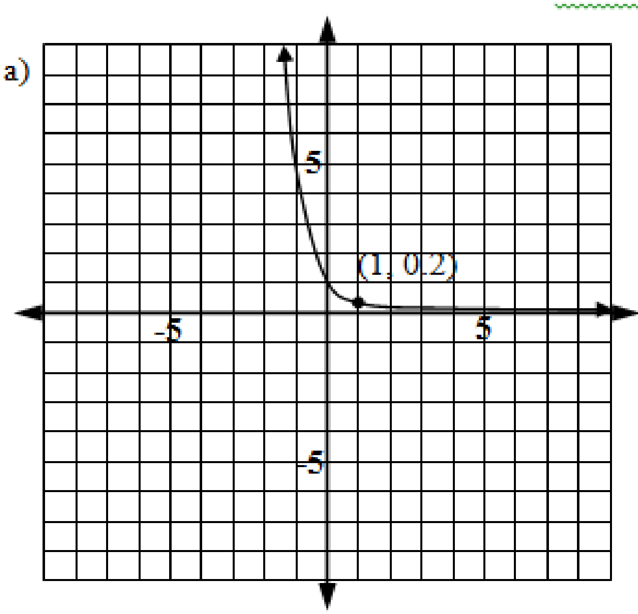
Here are some sample exponential graphs. What causes the graph to be steeper?



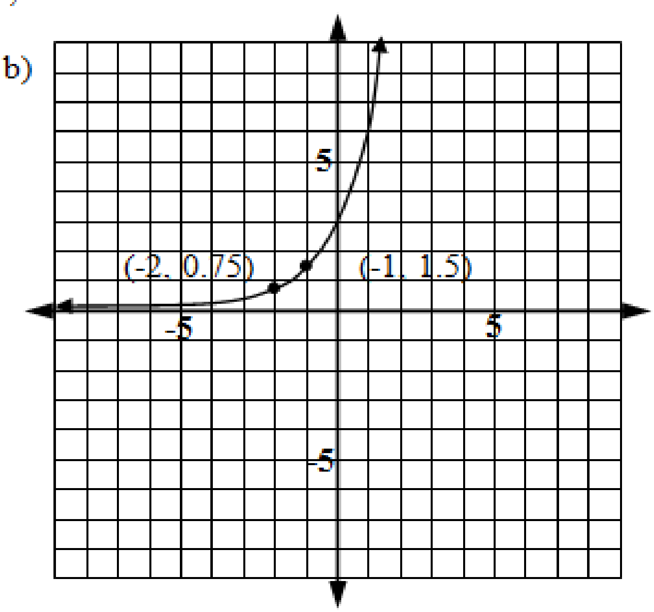


7 For each of the graphs below, write the correct exponential function. This means you need to determine the values of C and a for the formula $f(x) = C \cdot a^x$.

a)



b)



A special type of exponential function is used to model continuous growth.

NATURAL EXPONENTIAL FUNCTION

The function represented by

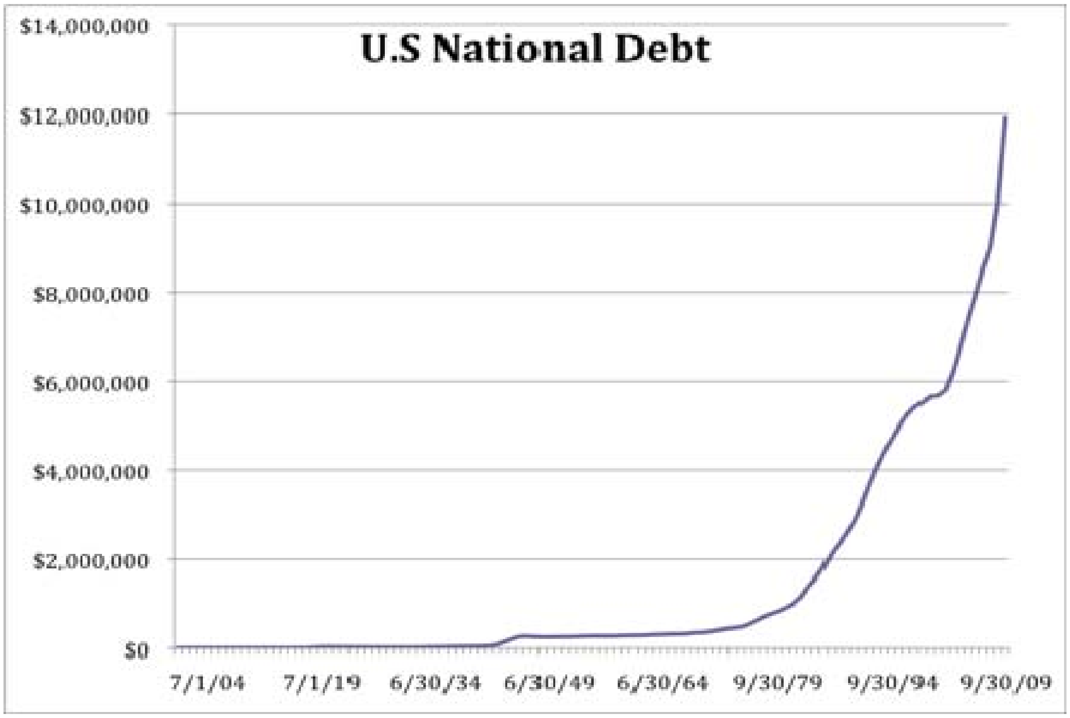
$$f(x) = e^x$$

is the **natural exponential function**, where $e \approx 2.71828$.

Money earning interest compounded continuously has the formula

$$\boxed{A = Pe^{rt}}$$

67. *Federal Debt* In ~~2003~~ 2010 the federal budget deficit was about ~~\$300 billion~~ 13.7 trillion. At the same time, 30-year treasury bonds were paying ~~4.95~~ 4.12% interest. Suppose that U.S. citizens loaned ~~\$300 billion~~ 13.7 trillion to the federal government at ~~4.95%~~ 4.12%. If the federal government waited 30 years to pay the entire amount back, including the interest, how much would it be? (**Source:** U.S. Treasury Department.)



81. *Modeling Population* (Refer to Example 7.) In 2000 the population of Arizona was 5 million and growing continuously at a rate of 3.1% per year.

(a) Write a function f that models Arizona's population in millions x years after 2000.

(b) Graph f in [0, 10, 1] by [4, 7, 1].

(c) Estimate the population of Arizona in 2010.

82. *Dating Artifacts* Radioactive carbon-14 is found in all living things and is used to date objects containing organic material. Suppose that an object initially contains C grams of carbon-14. After x years it will contain A grams, where

$$A = C(0.99988)^x.$$

(a) Let $C = 10$ and graph A over a 20,000-year period. Is this function an example of exponential growth or decay?

(b) How many grams are left after 5700 years? What fraction of the carbon-14 is left?