11.4 - Quadratic Formula

Note Title

QUADRATIC EQUATION

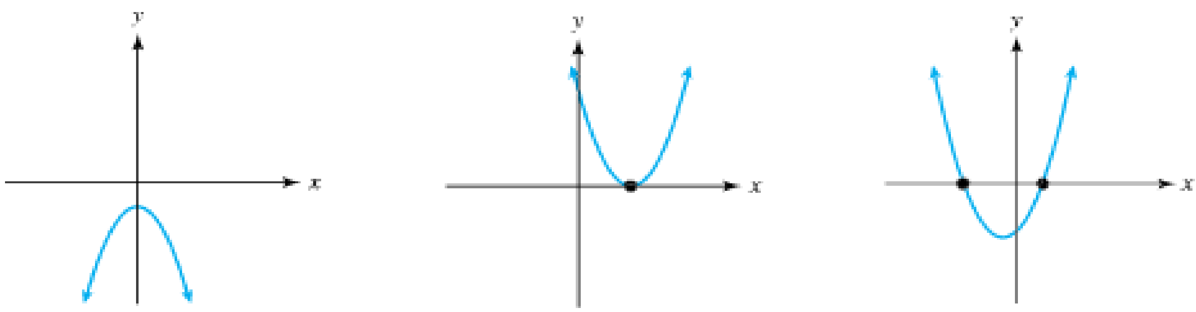
A **quadratic equation** is an equation that can be written as

$$ax^2 + bx + c = 0,$$

where a, b, and c are constants with $a \neq 0$.

This is the same as the x-intercepts of the graph (since).

There are 3 cases.



(a) No x-intercepts (b) One x-intercept (c) Two x-intercepts

There are many real world applications for solving quadratics.

To model the stopping distance of a car, highway engineers compute two quantities. The first quantity is the *reaction distance*, which is the distance a car travels from the time a driver first recognizes a hazard until the brakes are applied. The second quantity is *braking distance*, which is the distance a car travels after a driver applies the brakes. *Stopping distance* equals the sum of the reaction distance and the braking distance. If a car is traveling x miles per hour, highway engineers estimate the reaction distance in feet as $\frac{11}{3}x$ and the braking distance in feet as $\frac{1}{9}x^2$. See Figure 11.41. (**Source:** L. Haefner, *Introduction to Transportation Systems.*)

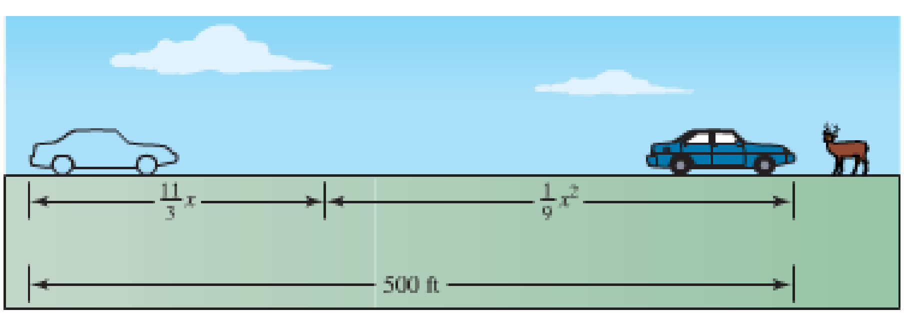


Figure 11.41 Stopping Distance

There are several techniques used to solve quadratic equations. (Can we just "isolate x"?) These include factoring, completing the square, square root property, and the quadratic formula.

We will only use the quadratic formula in this class, and it needs to be memorized.

QUADRATIC FORMULA

The solutions to $ax^2 + bx + c = 0$ with $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1. Solve the quadratic equations given.

a) $2x^2 + 11x - 6 = 0$

b) $36x^2 - 36x + 9 = 0$

c) $-3x^2+2x-1=0$

(2) If a car's headlights do not illuminate the road beyond 500 feet, estimate a safe nighttime speed limit x for the car by solving $\frac{1}{9}x^2 + \frac{11}{3}x = 500$.

The value under the $\sqrt{\quad}$ tells us how many solutions we'll have.

THE DISCRIMINANT AND QUADRATIC EQUATIONS

To determine the number of solutions to the quadratic equation $ax^2 + bx + c = 0$, evaluate the discriminant $b^2 - 4ac$.

1. If $b^2 - 4ac > 0$, there are two real solutions.
2. If $b^2 - 4ac = 0$, there is one real solution.
3. If $b^2 - 4ac < 0$, there are no real solutions; there are two complex solutions.

③ Determine the number of real solutions first, then find the solutions.

a) $5x^2 - 13x + 6 = 0$

b) $9x^2 + 12x + 4 = 0$

④ Solve

a) $x(x+3) = 3$

b) $2x = x(3 - 4x)$

Complex Roots

If your solution ends up with "−" in the radical, we call those <u>complex</u> <u>roots</u>.

$$\sqrt{-1} = i \quad , \qquad \text{so } \sqrt{-64} = 8i \text{ or } \sqrt{-40} = i\sqrt{40}$$

(5) Solve, including finding complex roots

a) $x^2 + 2x + 3 = 0$

b) $7x^2 - 2x + 4 = 0$

c) $2x^2 + x = 8$

d) $2x^2 = 2x(5-x) - 8$

111. *Modeling Water Flow* When water runs out of a hole in a cylindrical container, the height of the water in the container can often be modeled by a quadratic function. The data in the table show the height y in centimeters of water at 30-second intervals in a metal can that has a small hole in it.

Time	0	30	60	90
Height	16	11.9	8.4	5.3

Time	120	150	180
Height	3.1	1.4	0.5

These data are modeled by

$$f(x) = 0.0004x^2 - 0.15x + 16.$$

(a) Explain why a linear function would not be appropriate for modeling these data.

(b) Use the table to estimate the time at which the height was 7 centimeters.

(c) Use $f(x)$ and the quadratic formula to solve part (b).