

Borrow or Lend money?

These are flip sides of the coin.

- ① When you borrow money, someone else lends.
- ② when you lend money (deposits in bank are loans to bank), someone else borrows.

Interest = pay rent to use someone else's money.

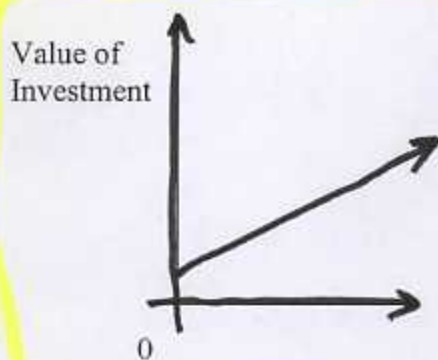
- ① when you borrow money you must pay interest (interest expense, or cash going out)
- ② when you lend money you receive interest (Interest Revenue, or cash received)

$$\left\{ \begin{array}{l} \text{The amount you} \\ \text{borrow or lend} \end{array} \right\} = \text{Principal} = \left\{ \begin{array}{l} \text{value of} \\ \text{loan or} \\ \text{investment} \end{array} \right\}$$

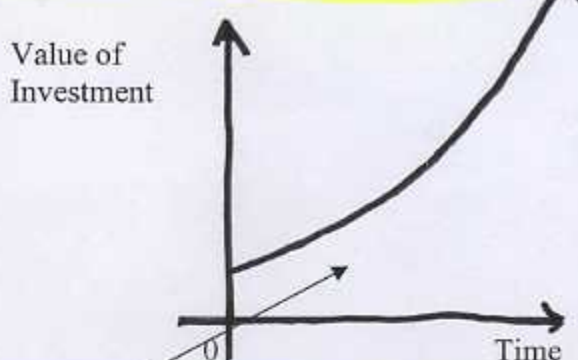
$$\left\{ \begin{array}{l} \text{If you borrow} \\ \text{money} \end{array} \right\} = \text{loan}$$

$$\left\{ \begin{array}{l} \text{If you lend} \\ \text{money} \end{array} \right\} = \text{investment}$$

SIMPLE INTEREST



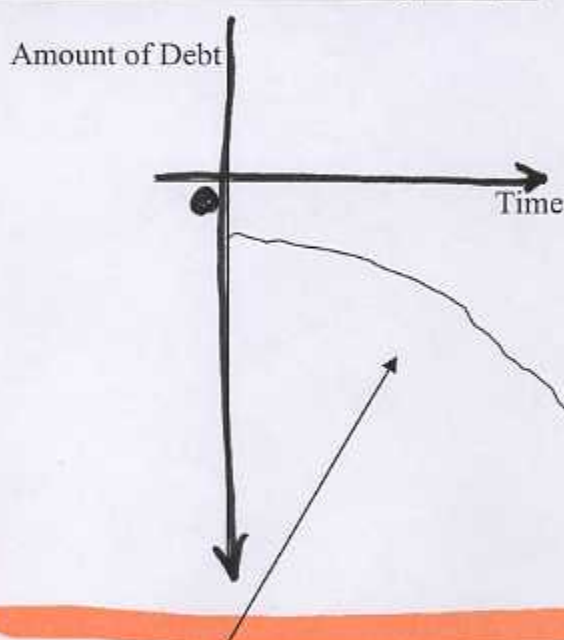
COMPOUND INTEREST works for you



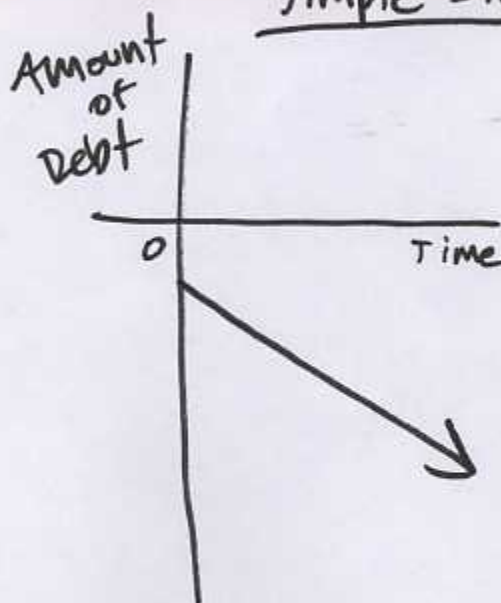
If you deposit \$, As time goes by your investment grows

THE POWER OF COMPOUND INTEREST WORKING FOR YOU

COMPOUND INTEREST works against you



Simple Interest



If you borrow \$ and pay it back all (principal and interest) at the end, as time goes by the amount you owe grows

THE POWER OF COMPOUND INTEREST WORKING AGAINST YOU

Fundamental Truth in Finance:

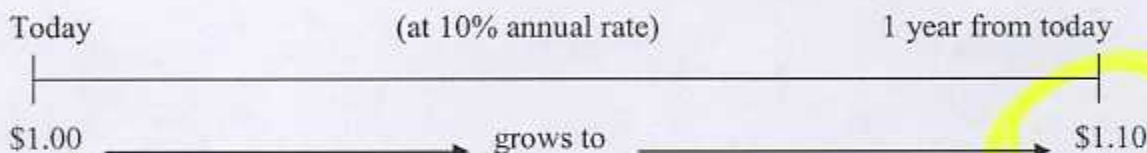
- A dollar received now is worth more than a dollar received later.
- This is true because of the ability of individuals to earn interest.

Because you can take the \$1⁰⁰ today and put it in the bank



The \$1 received today is worth more than a dollar received 1 year from now because you can invest the dollar and earn interest.

If you invest the \$1.00 in a bank account that earns 10% annual interest:



The terms we will use for this chapter are:

Present Value
(Interest going backwards)

Future Value
(Interest going forward)

The Present Value of the \$1⁰⁰ I receive today is \$1⁰⁰. But if I put it in the bank & earn 10% Annual Rate compounded once for the year, The Future Value of that \$1⁰⁰ will be \$1¹⁰. Therefore I would rather receive the \$1⁰⁰ today instead of waiting 1 whole year for \$1⁰⁰.

"The future value of \$1.00 is \$1.10"
"The Present Value of \$1.10 is \$1.00"

Chapter 9: Compound Interest

9.1) Compound Interest

a. Find compound interest

Simple Interest:

Interest paid on only the principal.

Compound Interest:

- 1) Interest paid on principal and past interest
- 2) Compound interest is calculated on any interest previously paid to the account in addition to the original principal

Compound Amount:

Total amount that an investment earns at maturity (FV)

MV = Maturity value
or
FV = Future value

Interest Earned:

Interest Earned = Compound Amount - Original Amount

**Works for both simple and compounded interest

Formula

Example:

Imagine you have deposited money in two different accounts:

- 1) \$1000 in an account that pays 10% interest compounded annually
- 2) \$1000 in an account that pays 10% simple interest
 - a. If you invest the money in each account for 4 years, what are the compound amounts for each account?
 - b. What is the difference between the two?
 - c. Which type of interest payments earns more?

$$x = \text{years} = 4$$

$$n = \text{number of compounding periods per year} = 1$$

$$i = 10\%$$

$$\text{Principal} = \text{Present value} = P = \$1000^{00}$$

$$\text{Compound amount} = \text{Maturity value} = \text{Future value} = FV = M$$

Find FV for 10% compounded Annually

Find FV for 10% simple interest

Find FV for 10% Compounded Annually

After Year 1

$$I = PRT = 1000 * .1 * 1 = \$100$$

$$FV \text{ at end of year 1} = 1000 + 100 = \$1100$$

After Year 2

$$I = PRT = 1100 * .1 * 1 = \$110$$

$$FV \text{ at end of year 2} = 1100 + 110 = \$1210$$

After Year 3

$$I = PRT = 1210 * .1 * 1 = \$121$$

$$FV \text{ at end of year 3} = 1210 + 121 = \$1331$$

After Year 4

$$I = PRT = 1331 * .1 * 1 = \$133.10$$

$$FV \text{ at end of year 4} = 1331 + 133.10 = \$1464.10$$

Find FV for 10% simple interest

$$I = PRT = 1000 * .1 * 4 = \$400$$

$$FV \text{ at end of year 4} = 1000 + 400 = \$1400$$

Future value = Maturity value = compound Amount = FV = M

FV w/ compound interest = 1464.10

- FV w/ simple interest = 1400.00

Compound Interest

= {Compound Interest earns this much more} \Rightarrow 64.10

Compound amount for \$1000 in an account that pays 10% interest compounded annually:

After year 1:

$$I = PRT = 1000 \times .10 \times 1 = \$100$$

$$\text{Value at end of 1 year} = 1000 + 100 = \$1100$$

After year 2:

$$I = PRT = 1100 \times .1 \times 1 = \$110$$

$$\text{Value at end of 2 years} = 1100 + 110 = \$1210$$

After year 3:

$$I = PRT = 1210 \times .1 \times 1 = \$121$$

$$\text{Value at end of 3 years} = 1210 + 121 = \$1331$$

After year 4:

$$I = PRT = 1331 \times .1 \times 1 = \$133.10$$

$$\text{Value at end of 4 years} = 1331 + 133.10 = \$1464.10$$

$$\text{Compound amount for compound interest} = \$1464.10$$

\$1000 in an account that pays 10% simple interest:

After year 1:

$$I = PRT = 1000 \times .10 \times 1 = \$100$$

$$\text{Value at end of 1 year} = 1000 + 100 = \$1100$$

After year 2:

$$I = PRT = 1000 \times .1 \times 1 = \$100$$

$$\text{Value at end of 2 years} = 1100 + 100 = \$1200$$

After year 3:

$$I = PRT = 1000 \times .1 \times 1 = \$100$$

$$\text{Value at end of 3 years} = 1200 + 100 = \$1300$$

After year 4:

$$I = PRT = 1000 \times .1 \times 1 = \$100$$

$$\text{Value at end of 4 years} = 1300 + 100 = \$1400$$

$$\text{Compound amount for simple interest} = \$1400$$

What is the difference between the two?

Maturity value with compounded interest =	\$1464.10
Maturity value with simple interest =	-\$1400.00
Compounded interest earned this much more =	\$64.10

Which type of interest payments earns more?

Account with compound interest always earns more!

Account with Compounded Interest:

$$\text{Interest} = \text{Compound Amount} - \text{Original Amount}$$

$$\text{Compounded Interest} = 1464.10 - 1000 = \$464.10$$

Account with Simple Interest:

$$\text{Interest Earned} = \text{Compound Amount} - \text{Original Amount}$$

$$\text{Simple Interest} = 1400 - 1000 = \$400$$

when we
compare simple
interest and
compound interest
our conclusions
are

b. Decide on a period of compounding

Period Interest Rate:

The interest rate earned for a stated period of time.

Hints for compounding interest:

- Compound Interest is often calculated more than one time a year
- Interest rates are usually given in annual terms
- This necessitates that you convert the annual interest rate to a period interest rate

Example:

If the 10% annual interest rate is compounded semiannually, what is the period interest rate?

$$\dot{i} = \text{Annual Interest Rate} = 10\%$$

$$n = \left\{ \begin{array}{l} \text{number of compounding} \\ \text{periods per year} \end{array} \right\} = 2$$

$$\left\{ \begin{array}{l} \text{Period} \\ \text{Interest} \\ \text{Rate} \end{array} \right\} = \frac{\dot{i}}{n} = \frac{.10}{2} = .05 \text{ or } 5\%$$

Example:

If the 10% annual interest rate is compounded quarterly, what is the period interest rate?

$$\dot{i} = \text{Annual Interest Rate} = 10\%$$

$$n = \left\{ \begin{array}{l} \text{number of compounding} \\ \text{periods per year} \end{array} \right\} = 4$$

$$\left\{ \begin{array}{l} \text{Period} \\ \text{Interest} \\ \text{Rate} \end{array} \right\} = \frac{\dot{i}}{n} = \frac{.10}{4} = .025 \text{ or } 2.5\%$$

Example:

If the 10% annual interest rate is compounded daily, what is the period interest rate?

$$\dot{i} = \text{Annual Interest Rate} = 10\%$$

$$n = \left\{ \begin{array}{l} \text{number of compounding} \\ \text{periods per year} \end{array} \right\} = 365 \text{ (for none leap years)}$$

$$\left\{ \begin{array}{l} \text{Period} \\ \text{Interest} \\ \text{Rate} \end{array} \right\} = \frac{\dot{i}}{n} = \frac{.10}{365} =$$

c. Use the formulas to find compound amount and compound interest

Exponent Lesson:

"Carrot"

~~2 * 2 * 2 * 2 = 16~~

$$2^4 \text{ or } 2^{^4}$$

$$= 2^4 = 2 * 2 * 2 * 2$$

$$= 4 * 2 * 2$$

$$= 8 * 2$$

$$= 16$$

$$\frac{.10}{365} = .000273972602739726 \approx .0274\%$$

Most saving Accounts
earn daily interest

Remember:

Compound amount for \$1000 in an account that pays 10% interest compounded annually:

After year 1:

$$I = PRT = 1000 \times .10 \times 1 = \$100$$

$$\text{Value at end of 1 year} = 1000 + 100 = \$1100$$

After year 2:

$$I = PRT = 1100 \times .1 \times 1 = \$110$$

$$\text{Value at end of 2 years} = 1100 + 110 = \$1210$$

After year 3:

$$I = PRT = 1210 \times .1 \times 1 = \$121$$

$$\text{Value at end of 3 years} = 1210 + 121 = \$1331$$

After year 4:

$$I = PRT = 1331 \times .1 \times 1 = \$133.10$$

$$\text{Value at end of 4 years} = 1331 + 133.10 = \$1464.10$$

Alternative method of calculating:

$$1000 \times (1 + .1) \times (1 + .1) \times (1 + .1) \times (1 + .1) = \$1464.10$$

$$FV = M = 1000(1 + .1)^4 = \$1464.10$$

$$FV = M = P \times (1 + i) \times (1 + i) \times (1 + i) \times (1 + i) \dots$$

$$\begin{aligned} FV = M &= 1000 * \left(1 + \frac{.1}{1}\right)^{(4 * 1)} \\ &= 1000 * \left(1 + \frac{.1}{1}\right)^4 \\ &= 1000 * (1 + .1)^4 \\ &= 1000 * (1.1)^4 \\ &= 1000 * 1.4641 \\ &= \$1464.10 \end{aligned}$$

Instead of ALL
This, you
could do this

OR EVEN
Better!!

OUR
Official
Formula
for this
chapter

Formula from book:

$$FV = M = P(1 + i)^n$$

M = Maturity Value (Future Value)

P = Principal or amount deposited in account

i = interest rate per period

n = Total number of periods

★ Note: we will ~~not~~ not use numbers from compound Interest Tables in text book. we will use formulas →

Formula that will work for all problems (no matter what the number of periods or period rate):

$$FV = M = P(1 + i/n)^{nx}$$

M = Maturity Value = Future Value

P = Principal amount invested

i = Annual interest rate

n = Number of periods in one year

x = Number of years

"Carrot" means exponent

$$FV = M = P * \left(1 + \frac{i}{n}\right)^{(n * x)}$$

All synonyms

FV = M = Future Value = Maturity Value = Compound Amount

P = Principal = present value

i = Annual Rate

n = number of compounding periods per year

x = Years

$x * n$ = total number of compounding periods

$\frac{i}{n}$ = period interest rate

in Excel we can use FV function

=FV (period interest rate, total number of compounding periods, -Present value)

★ Interest is paid each $\frac{1}{2}$ year

Example:

If you deposit \$2000 in an account that pays 10% annual interest compounded semiannually, what is the value of the account after 10 years? What is the total interest earned?

Step 1: List variables

$$P = 2000$$

$$i = 10\%$$

$$\rightarrow n = 2$$

$$x = 10$$

Step 2: use formula to find value of account at maturity

$$\begin{aligned} FV &= P * \left(1 + \frac{i}{n}\right)^{(n * x)} \\ &= 2000 * \left(1 + \frac{.10}{2}\right)^{(2 * 10)} \\ &= 2000 * \left(1 + \frac{.10}{2}\right)^{20} \\ &= 2000 * (1 + .05)^{20} \\ &= 2000 * (1.05)^{20} \\ &= 2000 * 2.6532977051442 \\ &= 5306.5954 \approx \$5306.60 \end{aligned}$$

Remember
order of
operations!

**the number from the formula is more accurate than the number from the Compound Interest Table

Step 4: How much interest was earned?

$$5306.60 - 2000 = 3306.60 \text{ Interest was earned!}$$

Difference between Compound Interest and Simple interest:

- With compounded interest relatively small differences in interest rates can add up over many compounding periods

How to distinguish the difference between Simple Interest Problems and Compounded Interest Problems:

Simple Interest Problems:

"Simple Interest" "Simple Interest Notes" "Discount Rate"

Compounded Interest Problems:

"Compounded Annually" "6% per Quarter" "Compounded Daily"

} Hint for
word
problems

★ Interest is paid each $\frac{1}{2}$ year

Example:

If you deposit \$2000 in an account that pays 10% annual interest compounded semiannually, what is the value of the account after 10 years? What is the total interest earned?

Step 1: List variables

$$P = 2000$$

$$i = 10\%$$

$$\rightarrow n = 2$$

$$x = 10$$

Step 2: use formula to find value of account at maturity

$$\begin{aligned} FV &= P * \left(1 + \frac{i}{n}\right)^{(n * x)} \\ &= 2000 * \left(1 + \frac{.10}{2}\right)^{(2 * 10)} \\ &= 2000 * \left(1 + \frac{.10}{2}\right)^{20} \\ &= 2000 * (1 + .05)^{20} \\ &= 2000 * (1.05)^{20} \\ &= 2000 * 2.6532977051442 \\ &= 5306.5954 \approx \$5306.60 \end{aligned}$$

Remember
order of
operations!

**the number from the formula is more accurate than the number from the Compound Interest Table

Step 4: How much interest was earned?

$$5306.60 - 2000 = 3306.60 \text{ Interest was earned!}$$

Difference between Compound Interest and Simple interest:

- With compounded interest relatively small differences in interest rates can add up over many compounding periods

How to distinguish the difference between Simple Interest Problems and Compounded Interest Problems:

Simple Interest Problems:

"Simple Interest" "Simple Interest Notes" "Discount Rate"

Compounded Interest Problems:

"Compounded Annually" "6% per Quarter" "Compounded Daily"

} Hint for
word
problems

9.2) Savings Accounts: Daily Interest

a. Define the Passbook account or Savings account and Time deposit accounts

Most banks pay interest, compounded daily.

Passbook accounts or Savings accounts:

- "Regular savings accounts"
- Money can be deposited or withdrawn at any time
- The Truth in Savings Act of 1991 mandates that interest on savings accounts be paid based on exact number of days
- Most of these accounts are safe because they are insured by the government FDIC Insured (Federal Deposit Insurance Corporation).
- Historically earn from annual 2.5% to 6% interest rate
- Book assumes a 3 ½% annual interest rate compounded daily

Time Deposit accounts:

- You earn a higher interest rate when you promise to leave your money for a longer period of time.
- You are required to leave the money in the account for a specified period of time.
- The time deposit account will typically earn more than a savings account.

CD or Certificate of Deposit account:

- A minimum amount of money must be left in for a minimum amount of time.
- CD accounts typically earn a higher rate of return than the time deposit account.

Remember:

Formula for finding the compound amount after making an investment:

$$FV = M = P(1 + i/n)^{nx}$$

M = Maturity Value = Future Value

P = Principal amount invested

i = Annual interest rate

n = Number of periods in one year

x = Number of years

Finding the interest earned:

Interest = compound amount - principal

b. Find interest compounded daily

Example:

Maria Carlota deposited \$4300 in a savings account paying $3\frac{1}{2}\%$ annual interest compounded daily on July 9 and withdrew the money on August 17. Find the value of her investment (money deposited) on August 17. How much interest did she earn?

Step 1: List Variables

$$\begin{aligned}P &= 4300 \\i &= 3\frac{1}{2}\% \Rightarrow .035 \\n &= 365\end{aligned}$$

Date invested = July 9

Date w/ draw = Aug 17

Days \$ in Account = 39

Step 2: Use formulas to solve for FV = M and interest

$$\begin{aligned}FV &= P \left(1 + \frac{i}{n}\right)^{(n * x)} \\&= 4300 * \left(1 + \frac{.035}{365}\right)^{(365 * .106849315068)} \\&= 4300 * \left(1 + .0000958904109589\right)^{39} \\&= 4300 * (1.0000958904109589)^{39} \\&= 4300 * 1.00374654756575 \\&= 4316.1101545327 \\&\approx \$4316.11 = \text{Future Value on Aug 17}\end{aligned}$$

$$\text{Interest Earned} = 4316.11 - 4300 = 16.11$$

Maria Carlota earned \$16.11 for leaving her money in the bank for 39 days.

Now what happens if we make withdrawals or deposits into the savings account? How much interest do we earn?

Example:

Maria Carlota deposited \$4300 in a savings account paying $3\frac{1}{2}\%$ annual interest compounded daily on July 9. Maria deposits an additional \$690 into the account on July 31. She withdrew the money on August 17. Find the value of her investment (money deposited) on August 17. How much interest did she earn?

****Treat each step deposit separately. Find the interest earned for each step.**

Step 1: The amount initially deposited was in the account for the entire time. Find the compound amount that resulted from the \$4300 that was left in the account for 39 days.

From the previous problem we know that the compounded amount = \$4316.11 ✓

Step 2: find the compound amount that resulted from the \$690 deposited on July 31.

First: Find the number of days that the investment was in the account

Invest day July 31
w/draw day Aug 17 } 17 days between these 2 dates

Second: use formula to find compounded amount

$$\begin{aligned} FV &= P * \left(1 + \frac{i}{n}\right)^{(n * x)} \\ &= 690 * \left(1 + \frac{.035}{365}\right)^{(365 * \frac{17}{365})} \\ &= 690 * \left(1 + \frac{.035}{365}\right)^{17} \\ &= 690 * 1.00163138810211 \\ &= 691.125657790 \approx \$691.13 \end{aligned}$$

Step 3: Add the two compound amounts together and find the interest earned

$$\begin{aligned} & (4316.11 + 691.13) - (4300 + 690) \\ &= 5007.24 - 4990 = \$17.24 \end{aligned}$$

The interest earned was
\$17.24

How do we calculate interest on Time Deposit Accounts?

Example:

Toni deposits \$20,000 in a 2-year time deposit account that earns an annual rate of 6% compounded daily. What is the value at maturity?

$$P = 20,000$$

$$X = 2 \text{ years}$$

$$i = 6\% \Rightarrow 0.06$$

$$n = 365$$

$$FV = P * \left(1 + \frac{i}{n}\right)^{n * X}$$
$$= 20,000 * \left(1 + \frac{0.06}{365}\right)^{(365 * 2)}$$

$$= 20,000 * 1.1274857323$$

$$= 22,549.7146467175$$

$$= \$22,549.71$$

The value at maturity (Future value)
is \$22,549.71

$$M = 20,000(1 + .06/365)^{365*2} = 20,000 * 1.127485731 = \$22,549.71$$

$$\text{Interest Earned} = \$22,549.71 - \$20,000 = \$2,549.71$$

Toni earned \$2,549.91 for leaving her money in the bank for 2 years.

c) Inflation and the CPI

Inflation:

- Continuous rise in the price of goods and services
- Increase in inflation means you have less purchasing power with a given amount of dollars
- Table in textbook

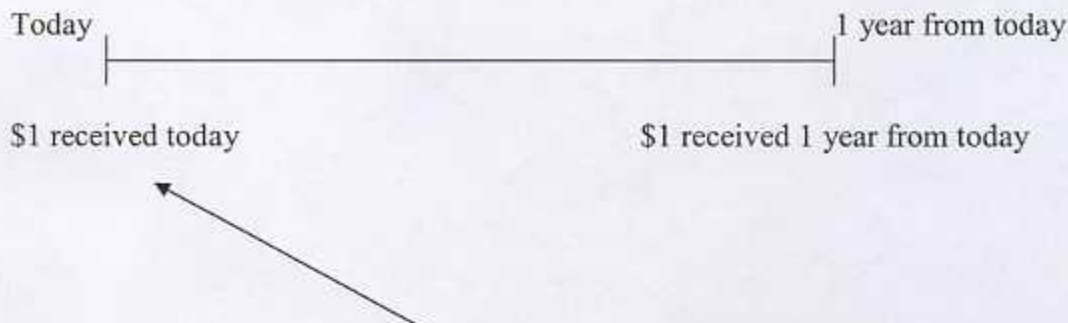
CPI (Consumer Price Index):

- A measure to track inflation
- Take average basket of goods and look to see how much the price increases over time

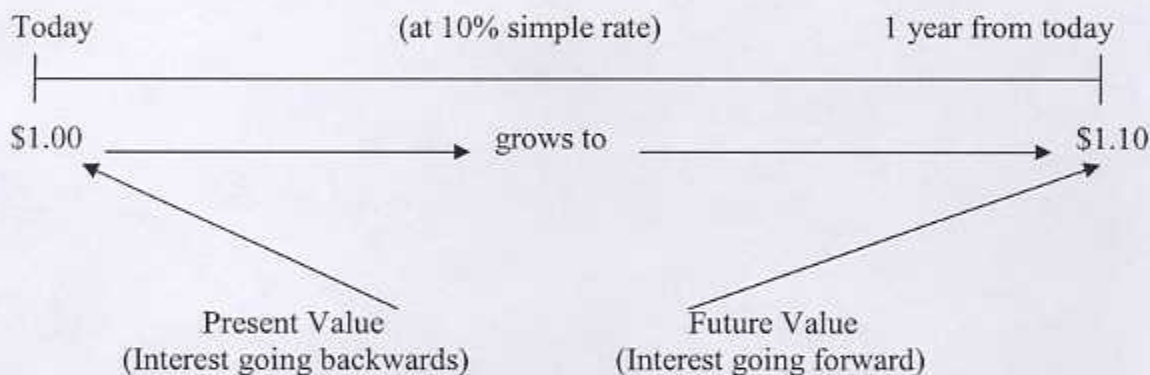
9.3) Future Value and Present Value

Fundamental Truth in Finance:

- A dollar earned now is worth more than a dollar earned later.
- This is true because of the ability of individuals to earn interest.

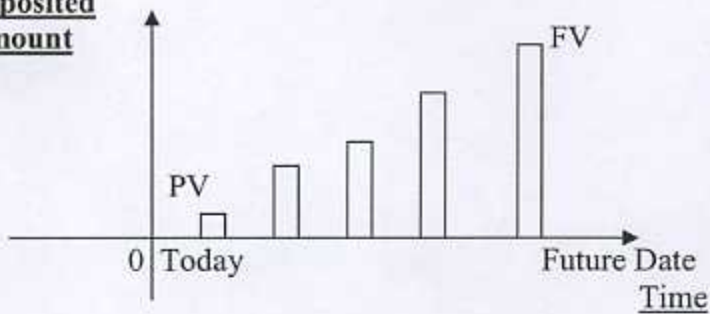


The \$1 received today is worth more than a dollar received 1 year from now because you can invest the dollar and earn interest.



The way to think of Present value is : "What do I have to invest today if I want to have a Future Value in a certain Amount of time?"

Deposited
Amount



Key
concepts!

Future Value (FV):

- "Interest going forward"
- The amount you receive at maturity
- Future Value = Maturity value
- The principal you invest is less than the maturity value
- The principal invested grows larger because of interest.
- As you move into the future, the interest gets larger
- As the interest gets larger, the maturity value or "future value" gets larger
- The maturity value or "future value" is growing because the interest is getting larger
- As you move into the future, the maturity value increases
- As you move into the future, the future value gets larger

Formula:

$$FV = M = P(1 + i/n)^{n \cdot x}$$

M = Maturity Value = FV

P = Principal or amount deposited in account = PV

i = interest rate per period

n = total number of periods

x = Number of years

Present Value (PV):

- "Interest going backwards"
- The amount that you must deposit today in order to receive the maturity value at a future time.
- If you know the amount you want in the future you can use the present value calculation to figure out how much you need to deposit today.
- Present Value = Principal funds deposited today
- Present value is like "interest in reverse."

Formula:

$$FV = M = P(1 + i/n)^{n \cdot x} \text{ becomes}$$

$$M/(1 + i/n)^{n \cdot x} = P$$

$$M \cdot 1/(1 + i/n)^{n \cdot x} = P$$

M = Maturity Value = FV

P = Principal or amount deposited in account = PV

i = interest rate per period

n = total number of periods

Key
concepts!

$$\left\{ \begin{array}{l} \text{Present} \\ \text{value} \end{array} \right\} = P = \frac{FV}{\left(1 + \frac{i}{n}\right)^{n \cdot x}}$$



How much
you have to
deposit today
in order to
get a certain
Future Value

FV = M = Future value

P = Present value

i = Annual Rate

n = number of compounding periods
per year

x = Years

x * n = total number of compounding
periods per year

$\frac{i}{n}$ = Period Interest Rate

In Excel we can use PV function

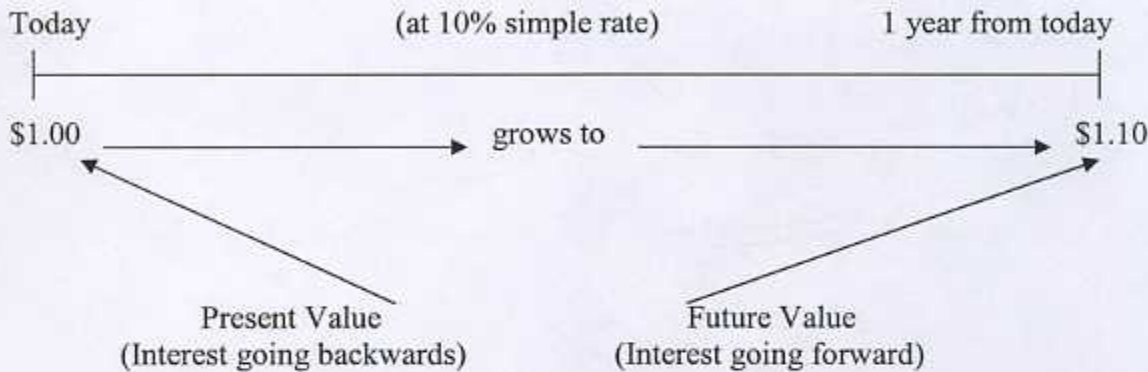
$$=PV\left(\text{period Interest Rate}, \text{total number of compounding periods}, -\text{Future value}\right)$$

$$M * 1/(1 + i/n)^{n*x} = P$$

Future Value amount

Present Value factor number

Present Value amount
(amount deposited)



"The future value of \$1.00 is \$1.10"

"The Present Value of \$1.10 is \$1.00"

Example:

If you invest \$1.00 at a 10% interest rate compounded annually, what is the future value after one year?

$$\begin{aligned}
 P &= 1 \\
 i &= .1 \\
 n &= 1 \\
 x &= 1 \\
 FV &= P \left(1 + \frac{i}{n}\right)^{n*x} \\
 &= 1 \left(1 + \frac{.1}{1}\right)^{1*1} = 1 (1.1)^1 = 1 * 1.1 = \$1.10
 \end{aligned}$$

Example:

If you wanted to find how much to invest now to receive \$1.10 one year from now, how much would you have to invest?

$$\begin{aligned}
 FV &= 1.10 \\
 i &= .1 \\
 n &= 1 \\
 x &= 1 \\
 PV &= \frac{FV}{\left(1 + \frac{i}{n}\right)^{n*x}} \\
 &= \frac{1.10}{\left(1 + \frac{.1}{1}\right)^{1*1}} = \frac{1.1}{(1+.1)^1} = \frac{1.1}{1.1} = \$1.00
 \end{aligned}$$

Example: If you wanted to have \$100,000 for your daughter's college education in 18 years, how much would you have to invest today if you could earn an annual rate of 12% compounded monthly?

$$FV = 100,000$$

$$X = 18$$

$$n = 12$$

$$i = 12\% \Rightarrow .12$$

$$P = ?$$

$$P = \frac{FV}{\left(1 + \frac{i}{n}\right)^{(n * X)}}$$

$$= \frac{100,000}{\left(1 + \frac{.12}{12}\right)^{(12 * 18)}}$$

$$= \frac{100,000}{(1 + .01)^{(216)}}$$

$$= \frac{100,000}{(1.01)^{216}}$$

$$= \frac{100,000}{8.57860629893636} = 11,656.90515630$$

If we wanted to have 100,000 for our daughter in 18 years and we could earn 12% compounded monthly, we would invest \$11,656.91

Example: If you wanted to have a million dollars in 40 years when you retire and you could earn an annual rate of 10% compounded monthly, how much would you have to invest today?

$$\begin{aligned}FV &= 1,000,000 \\i &= 10\% \Rightarrow .1 \\X &= 40 \\n &= 12 \\P &= ?\end{aligned}$$

$$\begin{aligned}P &= \frac{FV}{\left(1 + \frac{i}{n}\right)^{(n \cdot X)}} \\&= \frac{1,000,000}{\left(1 + \frac{.1}{12}\right)^{(12 \cdot 40)}} \\&= \frac{1,000,000}{(1.008\bar{3})^{480}} \\&= \frac{1,000,000}{53.7006631743289} \\&= 18621.74 \overline{39578668} \\&\approx \$18,621.74\end{aligned}$$

If I want to be a millionaire ^{in 40 years} & I can earn 10% compounded monthly, I would have to invest \$18,621.74 today

Juan needs to invest \$4083.48 today so she can have \$6000 in 5 years from now in order to buy the Morgan horse.

Vicki Frederick owns a restaurant worth \$125,000. She is confident that its value will increase at an annual rate of 16%, compounded semi annually for the next four years.

1) Find the future value of the restaurant in four years.

$$P = 125,000$$

$$i = 16\% = .16$$

$$n = 2$$

$$x = 4$$

$$\begin{aligned} FV &= P \left(1 + \frac{i}{n} \right)^{n(x)} \\ &= 125000 * \left(1 + \frac{.16}{2} \right)^{(2 * 4)} \\ &= \$ 231,366.28 \end{aligned}$$

The value of the restaurant in 4 years is \$231,366.25

- 2) If she sells the restaurant today, she will use the proceeds from the sale to invest in a bank account that pays 8% compounded quarterly. What selling price would she have to receive today in order to have the same Future value in four years (\$231,366.25)? (Opportunity cost problem).

The trick is to find the present value of the amount \$231,366.25 using the interest rate of the investment (8%) she would make if she had the money today (sold the business).

$$FV = 231,366.28$$

$$i = 8\% \Rightarrow .08$$

$$n = 4$$

$$X = 4$$

$$\begin{aligned} P &= \frac{FV}{\left(1 + \frac{i}{n}\right)^{(n * X)}} \\ &= \frac{231,366.28}{\left(1 + \frac{.08}{4}\right)^{(4 * 4)}} \\ &= \$168,537.80 \end{aligned}$$

Vicki Frederick should not sell the restaurant for less than \$168,538.74. If she were offered less than this amount, she would be better off keeping the restaurant. If she was offered more than this amount she would be better off selling the restaurant and investing in a bank account that pays 8% compounded quarterly.

**The key concept here is that you use the interest rate on the business to calculate the future value. You then use the interest rate on the bank account (alternative investment) to calculate the present value.