Math 163
Fall 2023
Assessment 7
Dusty Wilson

Name: key

Would you rather have unlimited bacon and no video games or unlimited video games and no video games?
Matt's question to Respawn Index
circa 2014

No work = no credit

1. Warm-ups

   (a) (1 point) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   (b) (1 point) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

   (c) (1 point) $\sum_{n=0}^{\infty} 3x^n$

2. (1 point) How would you respond to Matt's "Would you rather" question (above) Answer using complete English sentences.

   This is ridiculous, but the answer is easy for me: Bacon! I don't like video games.

3. (8 points) Find a power series expansion for $\frac{7x}{8+x^2}$ AND where the series converges.

$$\frac{7x}{8+x^2} = \frac{7x}{8} \cdot \frac{1}{1-\left(-\frac{x^2}{8}\right)}$$

$$= \sum_{n=0}^{\infty} \frac{7x}{8} (-1)^n \frac{x^{2n}}{8^n}$$

$$= \sum_{n=0}^{\infty} \frac{7(-1)^n x^{2n+1}}{8^{n+1}}$$

$$\left|\frac{x^2}{8}\right| < 1 \Rightarrow |x^2| < 8 \Rightarrow |x| < \sqrt{8}$$

on $-\sqrt{8} < x < \sqrt{8}$

4. (2 points) Simplify (a.) $\frac{1000!}{999!}$ and (b.) $\frac{n}{n!}$.

(a.) 1000 (b) $\frac{1}{(n-1)!}$

5. (4 points) Write $1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$ using sigma/summation notation.

$$\sum_{n=0}^{\infty} (-1)^n (n+1) x^n \quad \text{OR} \quad \sum_{n=1}^{\infty} (-1)^{n-1} n x^{n-1}$$

6. (8 points) Use a power series to approximate the definite integral $\int_0^{0.3} \frac{dx}{1+x^4}$ to six decimal places. Note: This requires using the first two non-zero terms of the series.

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{N=0}^{\infty} (-x^4)^N = \sum_{N=0}^{\infty} (-1)^N x^{4N}$$

$$\Rightarrow \int_0^{0.3} \frac{1}{1+x^4}\,dx = \left[\sum_{N=0}^{\infty} \int (-1)^N x^{4N}\,dx\right]_0^{0.3}$$

$$= \left[\sum_{N=0}^{\infty} \frac{(-1)^N x^{4N+1}}{4N+1}\right]_0^{0.3}$$

$$= \sum_{N=0}^{0.3} \frac{(-1)^N (0.3)^{4N+1}}{4N+1}$$

$$\approx 0.3 - \frac{0.3^5}{5}$$

$$= 0.299514$$

7. (4 points) Suppose you want to find the coefficients of the power series $\sum_{n=0}^{\infty} c_n x^n$. You know that $c_0 = 1$, $c_1 = 1$ and $c_n = c_{n-2} + c_{n-1}$ for $n \geq 2$. Write out the the first 10 terms of the series beginning with $1 + 1x + \ldots$

$$1 + 1x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6$$

$$+ 21x^7 + 34x^8 + 55x^9 + \ldots$$