100	90s	80s	70s	60s	<60
|	||||	~~||||~~ |||	~~||||~~ ||	|||	~~||||~~ ||

Math 163
Fall 2023
Assessment 6
Dusty Wilson

max 100

$\bar{x} = 73.8\%$

med = 77.9%

Name: Key

He knew he couldn't tell stories, that he always included extraneous details and tangents that interested only him.
John Green
1977 - present (American author)

No work = no credit

1. (3 points) Warm-ups

   (a) $\frac{d}{dx}\sin(x^2) =$ $2x\cos x^2$ (b) $\frac{\partial}{\partial x}\arctan(x) =$ $\frac{1}{1+x^2}$ (c) $\frac{\partial}{\partial y}y\sin(x^2) =$ $\sin x^2$

2. (1 point) What does it mean to "go off on a tangent"? Answer using complete English sentences.

   Starting off at a related point/topic & then talking about topics only slightly related is known as going off on a tangent.

3. (8 points) Consider $z = 3x + 4y^2$

   (a) (4 points) Find the tangent plane (linear approximation) at the point $A(2, 3)$

   $z(2,3) = 3(2) + 4(3)^2 = 6 + 36 = 42$

   $z_x(2,3) = 3$

   $z_y(2,3) = 8(3) = 24$

   $z - 42 = 3(x-2) + 24(y-3)$ ← tangent plane

   (b) (4 points) If (x, y) changes from $A(2, 3)$ to $B(1.7, 3.1)$ compare the values of Δz and dz

   $\Delta z = z(1.7, 3.1) - z(2,3)$

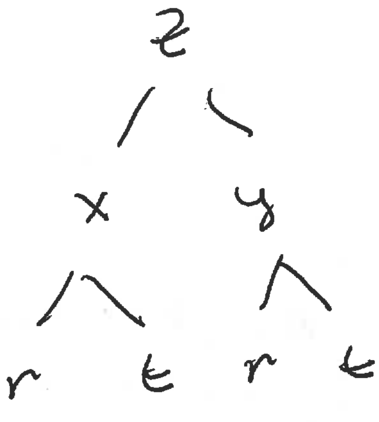
   $= 43.54 - 42$

   $= 1.54$

   $dz = 3(-0.3) + 24(0.1)$

   $= 1.5$

4. (4 points) If $z = 3x\sin(2y)$, $x = e^{rt}$, and $y = r\ln rt$, find $\frac{\partial z}{\partial t}$



$$\partial z = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= 3\sin 2y \cdot re^{rt} + 6x\cos(2y)\frac{r^2}{rt}$$

5. (14 points) Consider the function $g(x, y) = 2x^3 - 3xy^4$

(a) (4 points) Find the gradient at point $A(2, 1)$

$$\nabla g(x,y) = \langle 6x^2 - 3y^4, -12xy^3\rangle \Big|_{(2,1)} \quad \langle 21, -24\rangle$$

(b) (4 points) Interpret the (i.) direction and (ii.) magnitude of the gradient at point A

(i) g grows fastest @ $A(2,1)$ in the direction $\langle 21, -24\rangle$

(ii) The max ROC of g @ A is $\sqrt{1017} = 3\sqrt{113}$
as in the direction $\langle 21, -24\rangle$

(c) (2 points) At point A, in what direction $\langle x, y\rangle$ should we travel if we want our height on g to remain constant (NOT change)? Hint: There are multiple correct answers.

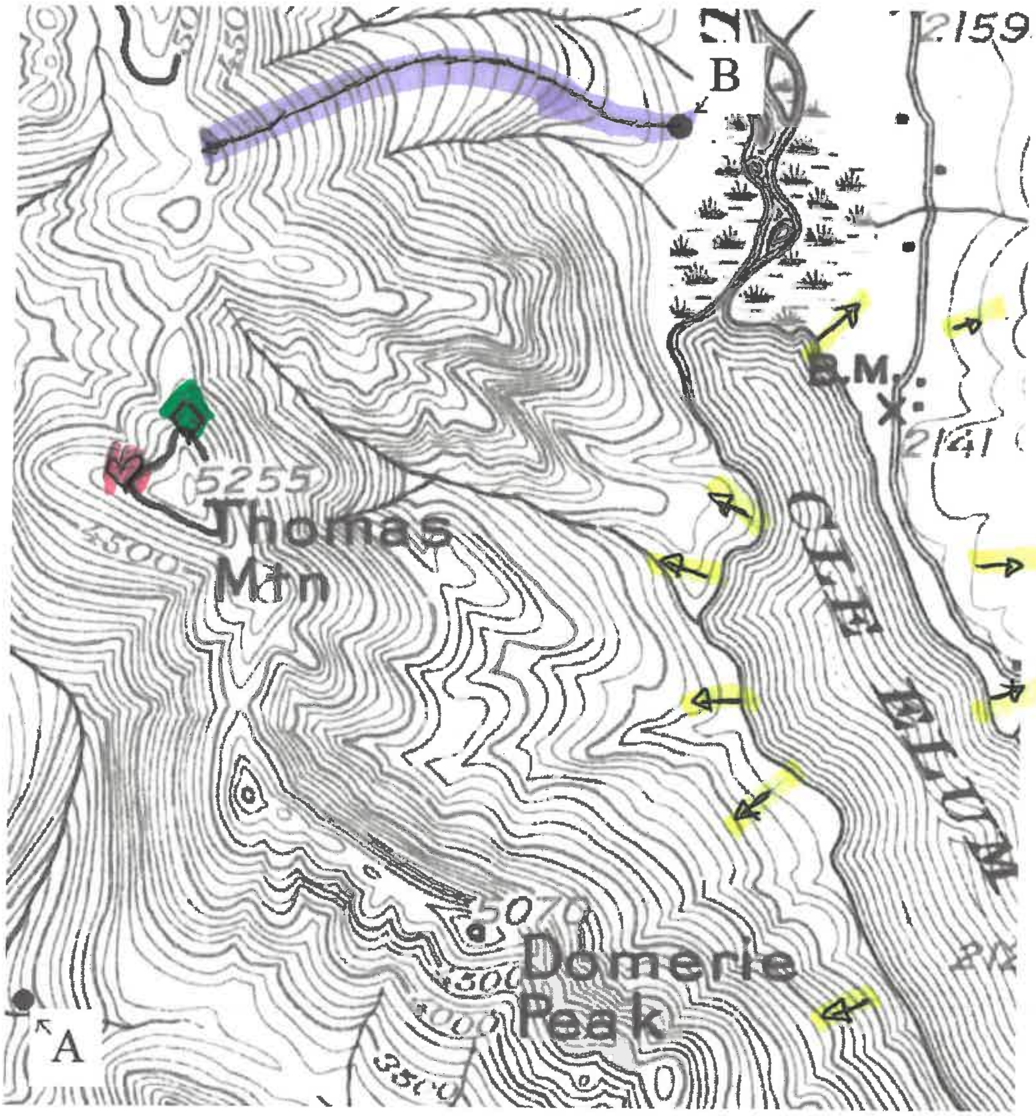
we want to be $\perp$ to the gradient
so go in the direction $\langle 24, 21\rangle$ because
$\langle 24, 21\rangle \cdot \langle 21, -24\rangle = 0$

(d) (4 points) Find the directional derivative of g at point A in the direction of the vector $\vec{v} = \langle 8, 15\rangle$

$\vec{u} = \frac{1}{17}\langle 8, 15\rangle$

$\langle 21, -24\rangle \cdot \langle \frac{8}{17}, \frac{15}{17}\rangle = \frac{-192}{17}$

6. (4 points) Consider the contour plot (topographical map) of the mountains near Snoqualmie Pass where $z = f(x, y)$ gives the altitude in feet at a point (x, y) where x and y have the traditional orientation. The solid black line(s) show level curves at 5,000 feet.



(a) (1 point) On the contour plot, clearly sketch at least 5 possible gradient vectors near Lake Cle Elum

(b) (1 point) On the contour plot, clearly mark with a $\diamond$ the point(s) of the level curve near Thomas Mtn where $f(x, y) = 5000$ at which $f_x = 0$ and $f_y < 0$

(c) (1 point) On the contour plot, clearly mark with a $\heartsuit$ the point(s) of the level curve near Thomas Mtn where $f(x, y) = 5000$ at which the slope is smallest ($|\nabla f|$ is small).

(d) (1 point) Beginning at point B, clearly sketch the path of steepest ascent.