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|-----|-----|-----|-----|-----|------|
| 100 | 90s | 80s | 70s | 60s | < 60 |
| 111 | 111 | 111 | 111 | 111 | 111  |

Name: \_\_\_\_\_

Key

Math 163

Fall 2023

Assessment 5

Dusty Wilson

$$\bar{x} = 68.7\%$$

$$\text{med} = 67.7\%$$

No work = no credit

If there is anything that can bind the mind of man to this dreary exile of our earthly home and can reconcile us with our fate so that one can enjoy living,—then it is verily the enjoyment of the mathematical sciences and astronomy.

Johannes Kepler

1571 - 1630 BC (Swiss astronomer)

1. (3 points) Warm-ups

(a)  $\frac{d}{dx} \tan(x) = \sec^2 x$

(b)  $\frac{\partial}{\partial x} \arctan(x) = \frac{1}{1+x^2}$

(c)  $\frac{\partial}{\partial x} \sqrt{y} = 0$

2. (1 point) What did Kepler look to (above) that brought him pleasure? Answer using complete English sentences.

Kepler found pleasure in math & science.

3. (4 points) Consider  $f(x, y) = 2x^2 \sin(y^4) + 5x - 6y + 7$

(a) (1 point) Find  $f_x(x, y) = 4x \sin(y^4) + 5$

(b) (2 point) Find  $\frac{\partial^2 f}{\partial x \partial y}$

$$\frac{\partial f}{\partial y} = 8x^2 y^3 \cos(y^4) - 6$$

$$\frac{\partial^2 f}{\partial x \partial y} = 16xy^3 \cos(y^4)$$

4. (4 points) Use the contour plot of the Island of Hawai'i to answer the following:

- (a) (1 point) Estimate the height of the highest point on Mauna Kea

About 14000 ft

- (b) (1 point) At which point is the terrain steepest? A, B, or D? Why?

"A" because the contours are closer together.

- (c) (1 point) Suppose you were hiking along path C. What can you say about your altitude?

The path stays @ a fixed altitude of 2000 ft.

- (d) (1 point) Explain what is happening to the altitude (height) <sup>around</sup> at point B.

B is @ a saddle point. It goes down to E-W, but up to the peaks N-S.

5. (19 points) Consider  $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$  on  $-\frac{\pi}{2} < t < \frac{\pi}{2}$

- (a) (1 point) If  $\vec{r}(t)$  includes  $(1, 0, 0)$ , what is  $t$ ? Use this value of  $t$  in subsequent parts of this question.

$$t = 0$$

- (b) (4 points) Find and fully simplify  $|\vec{r}'(t)|$

$$\vec{r}'(t) = \langle -\sin t, \cos t, -\tan t \rangle$$

$$|\vec{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$$

$$= \sqrt{1 + \tan^2 t}$$

$$= \sqrt{\sec^2 t}$$

$$= \sec(t)$$

- (c) (4 points) Find the curvature of  $\vec{r}(t)$  at the  $t$ -value found in (a).

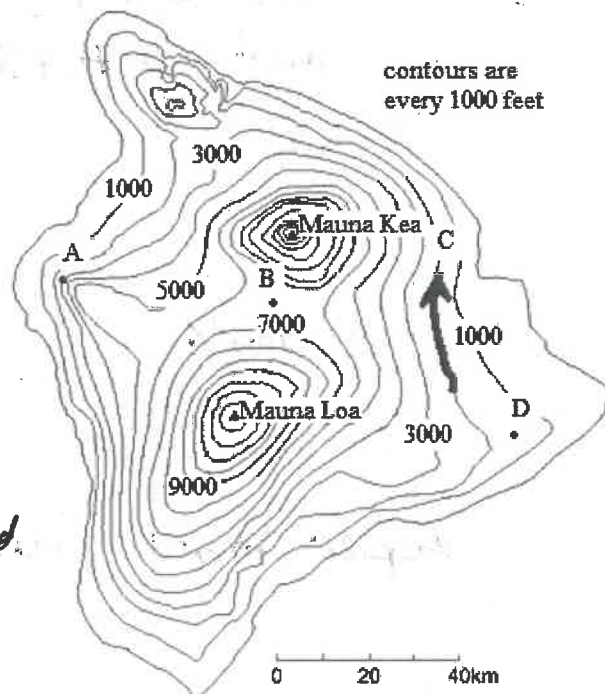
$$\vec{r}'(0) = \langle 0, 1, 0 \rangle \leftarrow \text{magnitude} = 1$$

$$\vec{r}''(t) = \langle -\cos t, -\sin t, -\sec^2 t \rangle$$

$$\Rightarrow \vec{r}''(0) = \langle -1, 0, -1 \rangle$$

$$\Rightarrow |\vec{r}'(0) \times \vec{r}''(0)| = |\langle 1, 0, 1 \rangle| = \sqrt{2}$$

$$\Rightarrow k = \frac{\sqrt{2}}{1^2} = \sqrt{2}$$



(d) (4 points) Find the tangential and normal components of acceleration at the  $t$ -value found in (a).

$$\vec{r}'(0), \vec{r}''(0) = 0$$

$$\Rightarrow a_T = 0$$

$$|\vec{r}'(0) \times \vec{r}''(0)| = \sqrt{2}$$

$$\Rightarrow a_N = \sqrt{2}$$

(e) (6 points) Find the tangential, normal, and binormal vectors at the  $t$ -value found in (a).

$$\vec{T}(t) = \frac{1}{\sec t} \langle -\sin t, \cos t, \tan t \rangle$$

$$= \langle -\frac{1}{2} \sin 2t, \cos^2 t, -\sin t \rangle \Big|_{t=0} \langle 0, 1, 0 \rangle$$

$$\Rightarrow \vec{T}'(t) = \langle -\cos 2t, -\sin 2t, -\cos t \rangle \Big|_{t=0} \langle -1, 0, -1 \rangle$$

$$\Rightarrow \vec{N}(t) = \frac{1}{\sqrt{2}} \langle -1, 0, -1 \rangle$$

$$\Rightarrow \vec{B}(t) = \frac{1}{\sqrt{2}} \langle -1, 0, +1 \rangle$$

1.  $\frac{1}{x^2} = x^{-2}$

2.  $\frac{1}{x^3} = x^{-3}$

3.  $\frac{1}{x^4} = x^{-4}$

4.  $\frac{1}{x^5} = x^{-5}$

5.  $\frac{1}{x^6} = x^{-6}$

6.  $\frac{1}{x^7} = x^{-7}$

7.  $\frac{1}{x^8} = x^{-8}$

8.  $\frac{1}{x^9} = x^{-9}$

9.  $\frac{1}{x^{10}} = x^{-10}$